Bias reduction on CPT-based correlations

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ABSTRACT: Cone penetration tests (CPTs) often provide benchmark data for geotechnical site characterisation through empirical correlations. These correlations are estimated from CPT data and soil property measurements. Both CPT data and soil property measurements contain random errors and systematic errors, making correlations less reliable. Reliability assessment of correlations is usually achieved by undocumented engineer-dependent judgement. A method is proposed for reducing the influence of both random and systematic errors on CPT-based correlations and their reliability.

1 INTRODUCTION

Cone penetration tests (CPT) often provide benchmark data for geotechnical site characterisation through empirical correlations, which inevitably involve uncertainty. The uncertainty arises from two sources, i.e. *random errors* (scatter) and *systematic errors* (bias). Accounting for both types of errors in the correlation assessment will lead to a more accurate approximation of the true value of the soil property, i.e. without random and systematic error. The true value of a soil property is usually defined by a "reference" value applicable to a selected geotechnical calculation model.

2 ERRORS IN MEASURED DATA

2.1 *Random errors: scatter*

Scatter is the result from random testing errors in measurements (noise) and inherent spatial variability in soil properties. Inherent spatial variability yields measurements fluctuating around the true value and, with sufficient data available, only has an influence on the variance of the parameter. Errors that affect the variance of a parameter only, are known as random errors.

2.2 *Systematic errors: bias*

Systematic error, or bias, is defined as the difference between the true value and the expected (estimated) value. Measurement errors that affect the expected value are called systematic errors. A major source for systematic errors in geotechnical parameters is (small scale) heterogeneity. Correlating CPT data with soil property data and with other in-situ test and laboratory test data will inevitably involve accounting for spatial variability, as any pairing of data will always be on material spaced at some distance.

3 CLASSICAL APPROACH TO DERIVING CPT-BASED CORRELATIONS

CPT data is usually plotted against, laterally adjacent, soil property data in an X-Y scatter plot, after which best fit correlations are determined ("post-regression"). Pairing of soil property measurements with CPT data introduces complexity, which includes:

- Relatively low soil property measurements may pair with relatively high CPT values and vice versa.
- There may be no CPT data available at the depths of the soil property measurements. This will be particularly evident for correlations of soil properties with (sparse) laboratory test data.
- A disadvantage of pairing soil property measurements with CPT data and subsequent correlation is that the paired CPT data values are used in the analysis only. The remaining CPT data is not taken into account.
- There may be significant lateral variability between the locations of the soil property measurements and the CPT data.
- The elevation of the locations where the soil property measurements and the CPT were taken may differ. Consequently, pairing data at equal depths may not be appropriate.

In post-regression, the CPT data are considered the independent variable, as it is the CPT data which is used to determine the best estimate of the soil property. Issues with post-regression include:

- Regression is typically based on a sparse data set, giving rise to confidence intervals.
- All data points are taken into account with the same weight, even though on the basis of engineering judgement some data points may be considered less reliable than others.

4 AN IMPROVED APPROACH TO DERIVING CPT-BASED CORRELATIONS

4.1 *Weighted least squares regression*

In geotechnical practice, a correlation estimate can be improved by first identifying strata with broadly similar geotechnical characteristics (and relevant for the geotechnical considerations of interest) and secondly determining the *weighted least* *squares regressions* (WLS) for each stratum through both the CPT data and the soil property measurements ("pre-regression"). Then, both regressions can be correlated, which accounts for the inherent spatial variability. Pre-regression mitigates the difficulty of sparse data, as regressions may be considered an "infinite" amount of data, i.e. data can be paired at any depth.

Trends through geotechnical data are usually assessed by means of ordinary least squares regression (OLS). This regression technique minimises the sum of squared residuals without any classification of data. By applying a weighted least squares regression on data, it is possible to incorporate the reliability of individual data points, by taking into account their weights. This technique minimises the weighted sum of squares:

$$\sum_{i=1}^{n} w_i (y_i - \vec{x}_i \cdot \beta)^2 \qquad (1)$$

(Note that if $w_i = 1$ for all data, then all data is of equal reliability, WLS is equal to the OLS). The weighting factors can be determined from the estimated systematic errors according:

$$w_j = 1 - \frac{\sqrt{\left[\left(\frac{\sum_{i=1}^{N} \varepsilon_{s\_lab(i)}}{\sum_{i=1}^{N} \max\left(\varepsilon_{s\_lab(i)}\right)}\right)^2 + \left(\frac{\sum_{i=1}^{N} \varepsilon_{s\_CPT(i)}}{\sum_{i=1}^{N} \max\left(\varepsilon_{s\_CPT(i)}\right)}\right)^2\right]}}{\sqrt{2}} \qquad (2)$$

where w_j = weight for each individual data point j; N = number of criteria; $\varepsilon_{s\_lab(i)}$ = systematic error in the laboratory measurement introduced by source i; $\varepsilon_{s\_CPT(i)}$ = systematic error in the CPT measurement introduced by source i; $\max(\varepsilon_{s\_lab(i)})$ = maximum possible systematic error in the laboratory measurement introduced by source i; and $\max(\varepsilon_{s\_CPT(i)})$ = maximum possible systematic error in the CPT measurement introduced by source i.

Using WLS makes it possible to consider all obtained data in the regression analysis (i.e. no data is eliminated), while directing the trend towards the true correlation, based on the more reliable data in the dataset.

The weights for WLS regression are determined by the systematic errors in each data set. Systematic errors will shift the expected value in a certain direction, with a certain magnitude. The magnitude and direction are not only site specific, but also depend on the source of the systematic error. The magnitude and direction of systematic errors in measurement data can be estimated from experience, soil catalogues and other test results. Depending on the soil property of interest, each measured parameter in the correlation function can be influenced by several systematic error sources.

4.2 *Confidence intervals*

Confidence intervals on the correlation can be determined using error propagation theory (Eq. 3):

$$\left(\frac{\partial z}{z}\right)^2 = \left(\frac{\partial x}{x}\right)^2 + \left(\frac{\partial y}{y}\right)^2 \tag{3}$$

where ∂x = error on measurement parameter x; ∂y = error on measurement parameter y; and ∂z = error on correlation of x and y.

5 ILLUSTRATION

The proposed methodology is illustrated by estimating the correlation of net cone resistance, q_n, with UU-triaxial undrained shear strength, c_u, being the reference strength recommended by API (2000) Main Text for axial pile capacity in clay. A common approach is to use a linear relation $c_u = q_n/N_k$, where q_n is the net cone resistance and N_k is the so called "cone factor". Reliability assessment of the N_k-factor is a difficult task, but crucial for accurate c_u derivation. The N_k-factor depends on sensitivity, overconsolidation, overburden stress, silt content and reference strength, such as UU-triaxial, vane shear test or pocket penetrometer undrained shear strength. Estimating both magnitude and direction of the systematic errors introduced by a number of sources will yield a more consistent N_k assessment. For UU-triaxial undrained shear strength and net cone resistance, the systematic errors could be introduced by the sources as presented in Table 1. Note that the value of the errors is site-specific and should only be considered example estimates.

Table 1. Suggested estimates of systematic errors on UU-triaxial undrained shear strength measurements and CPT net cone resistance measurements.

Error sources	Systematic error		
(1) Laboratory Value – UU Triaxial Compression Test c_u	$\varepsilon_{S\_UU\text{-}cu}$		
Undrained soil response	Practically undrained (10%)	Partially drained (25%)	Fully drained (50%)
Vertical Heterogeneity (consider shells, thin layers of sand/silt/peat, stratified etc.)	< 5% heterogeneity (5%)	5% to 50% heterogeneity (20%)	> 50% heterogeneity (40%)
(Macro) structure (consider cementation, fissures or other discontinuities)	None (5%)	Multiple, random (20%)	Oriented (40%)
Undrained shear strength ratio (UU accuracy typically reduces with increasing ratio)	$c_u/\sigma'_{v0} < 0.5$ (5%)	$0.5 < c_u/\sigma'_{v0} < 1$ (10%)	$c_u/\sigma'_{v0} > 1$ (25%)
Undrained shear strength c_u (UU has minimum "threshold" strength)	c_u >10 kPa (5%)	5< c_u <10 kPa (20%)	c_u < 5 kPa (40%)
Drilling, sampling and handling activities*) generate sampled undisturbed soil quality that matches "reference quality"	$\Delta e/e_0 < 0.04$ for OCR < 2 or $\Delta e/e_0 < 0.03$ for $2 \le$ OCR < 4 (-15%)	$0.04 \le \Delta e/e0 < 0.07$ for OCR < 2 or $0.03 \le \Delta e/e0 < 0.05$ for $2 \le$ OCR < 4 (40%)	$\Delta e/e0 > 0.07$ for OCR < 2 or $\Delta e/e0 > 0.05$ for $2 \le$ OCR < 4 (80%)

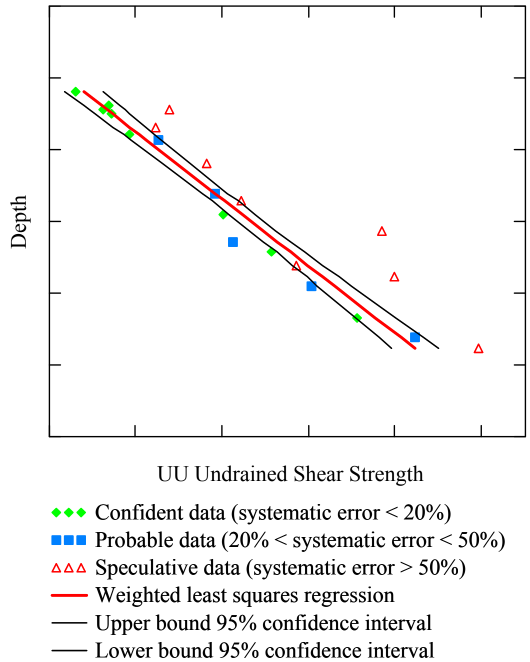
(2) In-Situ Value – CPT q_n	$\varepsilon_{S\ CPT\text{-}qn}$		
Undrained soil response	Practically undrained (5%)	Partially drained (15%)	Fully drained (50%)
Homogeneity (consider shells, thin layers of sand/silt/peat, stratified etc.)	< 5% heterogeneity (5%)	5% to 50% heterogeneity (20%)	> 50% heterogeneity (40%)
(Macro) structure (consider cementation, fissures or other discontinuities)	None (5%)	Multiple, random (20%)	Oriented (40%)
Net cone resistance q_n and Application Class AC (CPT has minimum "threshold" resistance; AC according to ISO, 2010; AC1 not normally feasible in practice)	AC 1 q_n>0 kPa or AC 2 q_n>100 kPa or AC 3 q_n>200 kPa (5%)	AC 1 10<q_n<30 kPa or AC 2 30<q_n<100 kPa or AC 3 70<q_n<200 kPa (15%)	Other conditions (25%)
Pore pressure measurement system fully saturated (consider loss of saturation due to penetration of unsaturated soil and/or dilative clays/sands; loss of saturation unlikely where in-situ equilibrium pore pressure >2000 kPa)	Confident or Probable with Q_t>10 or Speculative with Q_t>20 (5%)	Probable or Speculative with Q_t>10 (15%)	Speculative (25%)

*) Reduced quality usually gives lower than expected c_u values; chemical/biological alteration and high-quality practice usually give higher than expected c_u values; consider offshore drill string movements, mud pressure window, soil composition, strength sensitivity, void ratio comparisons etc.

A classification system for the data consisting of 3 rankings, being 1) Confident (ε_S < 20%); 2) Probable (20% < ε_S < 50%); and 3) Speculative (ε_S > 50%) is proposed. Ideally, correlation is based on confident data only. However, in geotechnical practice, only sparse data will be ranked confident (even as little as one or two data points). Therefore, it is necessary to assess the data so that an optimum correlation estimate can be achieved. This can be realised by applying WLS.

The proposed methodology is illustrated by comparing the common post-regression method with the weighted pre-regression method.

Consider a UU-triaxial undrained shear strength dataset (plotted versus depth, Fig. 1). The collection of squares, triangles and diamond shaped points represent the measured data, including systematic and random errors. The measured data is classified by means of weighting (equation 1). The WLS is represented by the red line and the two black hourglass-shaped lines represent the 95% confidence interval. A similar plot is made for the net cone resistance (Fig. 2).



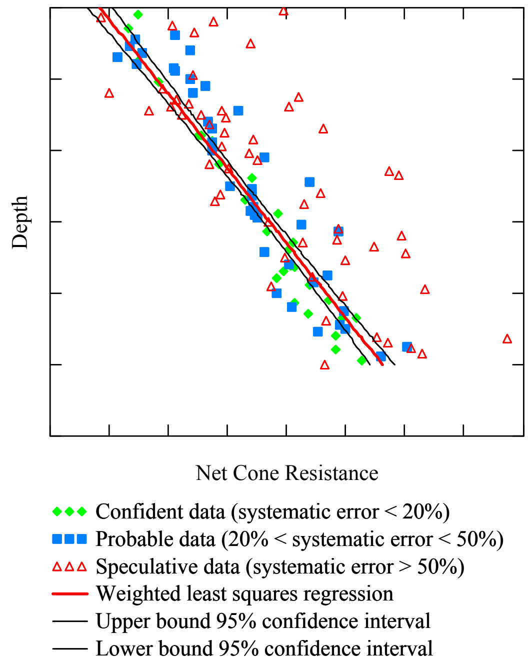


Figure 1. UU-triaxial undrained shear strength versus depth Figure 2. Net cone resistance versus depth

Figure 2 shows that the WLS regression pushes the correlation estimate towards the confident data, which is considered to have smaller systematic errors.

Subsequently, the weighted pre-regression from the UU-triaxial measurements is plotted against the weighted pre-regression from the net cone resistance measurements, as presented in Fig. 3.

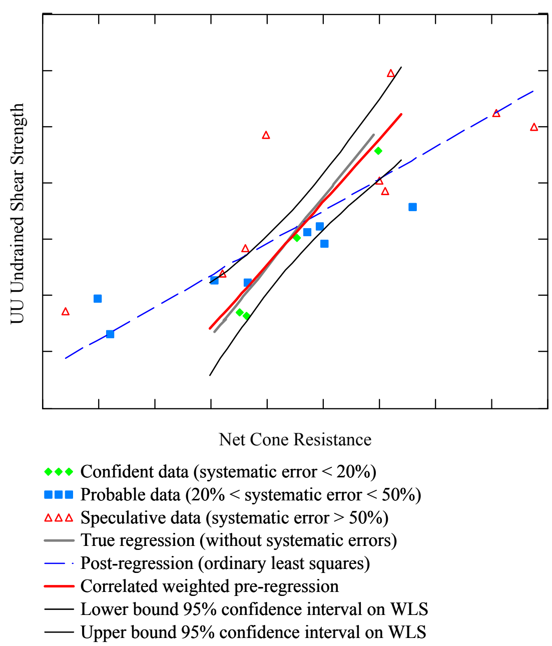


Figure 3. Correlation of weighted pre-regression

The (true) regression through the measurements without systematic errors is represented by the grey line. The dashed line represents the OLS through the scattered (paired) data (post-regression). The correlated weighted regressions are represented by the red line, indicating a better match with the true regression. The 95% confidence interval is determined using error propagation theory according Eq. 3.

6 CONCLUSIONS

Correlating soil property measurements with CPT measurements inevitably involves random errors and systematic errors. The work presented shows that by means of pre-regression, errors introduced by inherent spatial variability and small scale heterogeneity can be reduced. Subsequently, by estimating the systematic errors introduced by several sources, weighting factors can effectively be determined and used in weighted least squares regression techniques (WLS). Combining pre-regression and WLS results in more reliable CPT-based correlation estimates.

The presented error sources for correlation of net cone resistance and UU undrained shear strength serve as an example of application of the method. Similar criteria can be formulated for other correlations.

REFERENCES

API American Petroleum Institute (2000), "Recommended Practice for Planning, Designing and Constructing Fixed Offshore Platforms - Working Stress Design", API Recommended Practice 2A-WSD (RP 2A-WSD), 21st Edition. (With Errata and Supplement 1, December 2002, Errata and Supplement 2, October 2005, and Errata and Supplement 3, March 2008).