# Chapter 2
# Phase Relations

## 2.1 INTRODUCTION

Soils generally contain *soil grains*, *water* and *air*, which are known as the three phases. The relative proportions of these three phases play an important role in the engineering behaviour of the soils. Two extreme cases are *dry* soils and *saturated* soils, both having only two phases. Dry soils do not have water, and all the voids are filled by air. Saturated soils do not have air, and the voids are filled by water only. Very often, in geotechnical problems (e.g., earth works) and laboratory tests, it is necessary to compute the masses (or weights) and volumes of these three phases.

In this chapter, you will learn how to compute masses (or weights) and volumes of the soil grains, water and air in the soils. Let's define some simple terms and develop expressions relating them. These would be very useful in the phase relation calculations. It is important to *understand* (They are quite logical. You don't have to memorise them.) these definitions. They will appear in almost every chapter in this subject.

## 2.2 DEFINITIONS

Let's consider a soil mass shown in Fig. 2.1 (a), where all three phases are present. The soil grains, water and air are separated in Fig. 2.1 (b), which is known as the *phase diagram.* In the phase diagram, volumes are shown on the left and weights (or masses) are shown on the right.

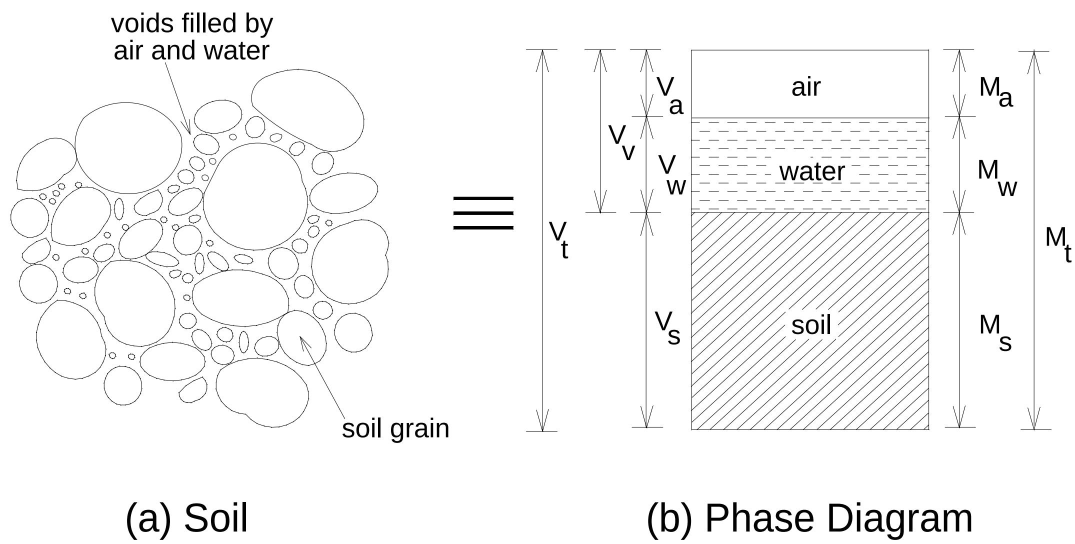


**Figure 2.1 Phase Relations of Soils**

*Water content* (w) is a measure of the amount of water present in the soil. It is defined as:

$$w = \frac{M_w}{M_s} \times 100\ (\%)$$

The natural water content for most soils would be well below 100%, but organic soils and some marine clays can have water contents greater than 100%.

The void content of a soil is expressed through two simple terms, *void ratio* (e) and *porosity*(n), defined as:

$$e = \frac{V_v}{V_s}$$

and

$$n = \frac{V_v}{V_t} \times 100\ (\%)$$

It should be noted that while e is expressed as a decimal number, n is traditionally expressed as a percentage ranging from 0 to 100%. Void ratios of sands may range from 0.4 to 1 and for clays these can vary from 0.3 to 1.5. For soft clays and organic soils, e can be even more.

> It is necessary to understand the definitions of the specific terms introduced in this chapter. The expressions relating these terms, including the ones in boxes, can be derived from the first principles within a few minutes. When carrying out phase computations, it is a good practice to go from the first principles.

The *degree of saturation* (S) is a measure of the void volume that is filled by water, expressed as a percentage ranging from 0 to 100. It is defined as:

$$S = \frac{V_w}{V_v} \times 100\ (\%)$$

For a completely dry soil S = 0%, and for a soil where the voids are completely filled with water (*saturated soil*) S = 100%. Soils below the water table are often saturated.

*Unit weight* ($\gamma$) of a soil is simply the weight per unit volume. However, because of the different phases present in the soil, several forms of unit weights are used in geotechnical engineering. The most common one is the *bulk unit weight* ($\gamma_m$), which is also known as *total*, *wet* or *moist* unit weight. It is the total weight divided by the total volume, and is written as:

$$\gamma_m = \frac{M_t}{V_t}$$

*Dry unit weight* ($\gamma_d$) is the unit weight of the soil when dry. Therefore, it can be written as:

$$\gamma_d = \frac{M_s}{V_t}$$

*Saturated unit weight* ($\gamma_{sat}$) is the bulk unit weight of a soil when it is saturated. *Submerged unit weight* ($\gamma'$) is the effective unit weight of a submerged soil, and is given by:

$$\gamma' = \gamma_{sat} - \gamma_w$$

where, $\gamma_w$ is the unit weight of water, which is 9.81 kN/m$^3$.

*Densities* ($\rho$) are similar to unit weights, except that mass, instead of weight, is used in the computations. Thus, bulk density ($\rho_m$), dry density ($\rho_d$), saturated density ($\rho_{sat}$) and submerged density ($\rho'$) can be defined in a similar manner. Density of water ($\rho_w$) is 1 g/cc, 1 t/m$^3$ or 1000 kg/m$^3$. You may remember that $\gamma = \rho g$.

*Specific gravity* of a soil grain ($G_s$) is the ratio of the density of *soil grain* to the density of water. It tells us how many times the soil grain is heavier than water. For most soils, $G_s$ varies in a very narrow range of 2.6 to 2.8. Nevertheless, there are exceptions where for mine tailings rich in minerals, we have measured specific gravity values as high as 3.8. For organic soils it can be as low as 2. In phase computations, if $G_s$ is not given, it is reasonable to assume a value in this range.

## 2.3 PHASE RELATIONSHIPS

All the terms introduced above (e.g., w, $\rho_m$) are ratios and thus do not depend on the amount of soil under consideration. In a homogeneous soil mass they should be the same anywhere. Let us consider a portion of the soil shown in Fig. 2.1a, where the volume of the soil grains is 1 unit volume. Using the terms defined above, the volumes and weights of the three phases can be determined as shown in Fig. 2.2.

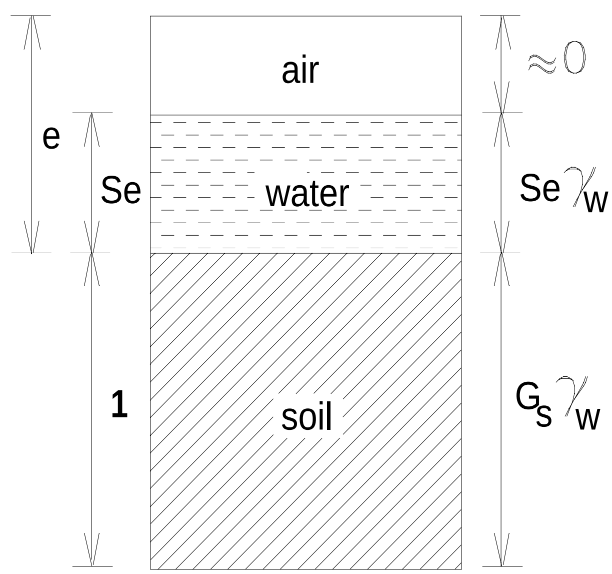


**Figure 2.2 Phase Relations when $V_s = 1$**

For $V_s = 1$, $V_v$ and $V_w$ are e and Se respectively. The weights are obtained multiplying the volumes by the appropriate unit weights.

Now let us develop some simple and very useful expressions for w, n, $\gamma_m$, etc. using Fig. 2.2.

Water content, when expressed as a decimal number, is:

$$w = \frac{M_w}{M_s} = \frac{Se\gamma_w}{G_s\gamma_w}$$

$$w = \frac{Se}{G_s}$$

Porosity, when expressed as a decimal number, is:

$$n = \frac{V_v}{V_t} = \frac{e}{1+e}$$

Bulk unit weight is:

$$\gamma_m = \frac{M_t}{V_t} = \frac{G_s + Se}{1+e}\gamma_w$$

An expression for saturated unit weight can be obtained by substituting S = 1 in the above expression (Note that w and S are expressed as decimal numbers in these expressions).i.e.,

$$\gamma_{sat} = \frac{G_s + e}{1+e}\gamma_w$$

## EXAMPLES

1. A cylindrical specimen of moist clay has a diameter of 38 mm, height of 76 mm and mass of 174.2 grams. After drying in the oven at 105°C for about 24 hours, the mass is reduced to 148.4 grams. Find the dry density, bulk density and water content of the clay. Assuming the specific gravity of the soil grains as 2.71, find the degree of saturation.

   Solution:

   Volume of the specimen = $V_t = \pi\,(1.9)^2(7.6) = 86.2\ \text{cm}^3$

   $M_t = 174.2$ g

   $M_s = 148.4$ g

   $\therefore \rho_d = 148.4/86.2 = 1.722\ \text{g/cm}^3$

   $\rho_m = 174.2/86.2 = 2.021\ \text{g/cm}^3$

   $w = M_w/M_s = (174.2-148.4)/148.4 = 0.174$ or 17.4%

$$\rho_d = \frac{G_s\rho_w}{1+e}$$

$$\therefore \quad e = \frac{(2.71)(1)}{1.722} - 1.0 = 0.574$$

$$w = \frac{Se}{G_s}$$

$$\therefore S = \frac{(0.174)(2.71)}{0.574} = 0.821 \text{ or } 82.1\%$$

2. Field density testing (e.g., sand replacement method) has shown bulk density of a compacted road base to be 2.06 t/$m^3$ with water content of 11.6%. Specific gravity of the soil grains is 2.69. Calculate the dry density, porosity, void ratio and degree of saturation.

   Solution:

$$w = \frac{Se}{G_s}$$

$\therefore$ Se = (0.116)(2.69) = 0.312

$$\rho_m = \frac{G_s + Se}{1 + e}\rho_w$$

$$\therefore 2.06 = \frac{2.69 + 0.312}{1 + e} \times 1.0$$

$\therefore$ e = 0.457

3. 5 kg of soil, at natural water content of 3%, is to be mixed with water to achieve water content of 12%. How much water would you add to the above soil?

   Solution:
   Let the mass of the dry soil be x kg.

$$w = 0.03 = \frac{5 - x}{x}$$

$\therefore$ x = 4.854 kg

At w = 12%, $M_w$ = (4.854)(0.12) = 0.582 kg
At w = 3%, $M_w$ = (4.854)(0.03) = 0.146 kg
$\therefore$ amount of water to add = 0.582 - 0.146 kg
= 436 g = 436 ml

4. Soil excavated from a borrow is being used to build an embankment. The void ratio of the soil at the borrow is 1.14 and the porosity of the compacted soil in the embankment is 40%. If the volume of excavation is 200,000 m$^3$, how many cubic metres of embankment can be built using this soil?

Solution:
Let's equate the volume of the soil grains (say $V_s$ = x), which should be the same in the borrow and the embankment.
At the borrow,

$$e = 1.14 = \frac{200{,}000 - x}{x}$$

$\therefore$ x = 93457.9 m$^3$

At the embankment,

$$n = 0.4 = \frac{V_v}{V_t} = \frac{V_t - 93457.9}{V_t}$$

$\therefore V_t$ = 155,763 m$^3$

5. The undisturbed soil at a given borrow pit is found to have the following properties: w = 15%; $\gamma_m$ = 19.04 kN/m$^3$; $G_s$ = 2.70. The soil from this borrow is to be used to construct a rolled fill having a finished volume of 38230 m$^3$. The soil is excavated by means of a shovel and dumped onto trucks having a capacity of 3.82 m$^3$ each. When loaded to capacity, these trucks are found to contain, on the average, a net weight of soil and water equal to 57.85 kN.

In the construction process, the trucks dump their load on the fill, the material is spread and broken up, after which a sprinkler adds water until the water content is equal to 18%. The soil and water are thoroughly mixed and then compacted until the dry unit weight is equal to 17.28 kN/m$^3$.

(a) Assuming that each load is a full capacity load, how many truck loads are required to construct the fill?
(b) What should be the volume of the pit that remains in the borrow area, after all the material required for the fill has been removed?
(c) How many litres of water will have to be added per truck load, assuming that the moisture lost by evaporation during excavation, haulage and handling is negligible?
(d) If the fill should become saturated at some time subsequent to the construction and does not change volume appreciably, what will be its saturation water content?

<u>Solution</u>:

At borrow pit:
$w = 15\%$
$\gamma_m = 19.04$ kN/m$^3$

In the truck:
$V_t = 3.82$ m$^3$
$M_t = 57.85$ kN
$M_s = x$ kN (say)

At the fill:
$V_t = 38230$ m$^3$
$w = 18\%$
$M_s = y$ kN (say)

(a) In the truck,

$w = 15\%$

$$\therefore \quad \frac{57.85 - x}{x} = 0.15$$

$\therefore$ x = 50.30 kN

At the fill,

$M_s = y = (17.28)(38230)$
$= 660,614.4$ kN

$\therefore$ No. of truck loads $= 660,614.4/50.30$
$= 13134$

(b) At the borrow pit,

$M_s = 660,614.4$ kN and $w = 15\%$
$\therefore M_w = (0.15)(660614.4)$
$= 99092.2$ kN
$\therefore M_t = 759,706.6$ kN
$\therefore V_t = 759706.6/19.04$
$= 39900$ m$^3$

(c) $M_s$ per truck load is 50.30 kN
The water content is to increase from 15% to 18%.
Therefore, the amount of water to add for every truck load is:
$= (50.30)(0.03)$
$= 1.509$ kN
$= 153.8$ litres

(d) At the compacted fill

$$\gamma_d = \frac{G_s \gamma_w}{1+e}$$

$$e = \frac{(2.70)(9.81)}{17.28} - 1 = 0.533$$

If saturated at the same void ratio,

$$w = \frac{Se}{G_s} = \frac{(1)(0.533)}{2.70} = 0.197 = 19.7\%$$

## PROBLEMS

1. Show that:

$$\gamma_d = \frac{\gamma_m}{1+w}$$

and

$$\gamma' = \frac{G_s - 1}{1+e}\gamma_w$$

2. A 75 mm (internal diameter) thin walled sampling tube is pushed into the wall of an excavation and a 200 mm long undisturbed specimen, weighing 1740.6 g, was obtained. When dried in the oven, the specimen weighed 1421.2 g. Assuming that the specific gravity of the soil grains is 2.70, find the void ratio, water content, degree of saturation, bulk density and dry density.
*(Ans. 0.679, 22.5%, 89.5%, 1.970 g/cm$^3$, 1.608 g/cm$^3$)*

3. Find the weight of a 1.2 m$^3$ rock mass, having a porosity of 1%. Assume that the specific gravity of the rock mineral is 2.69.
*(Ans. 31.35 kN)*

4. A soil sample has the following characteristics: w = 18.5%, $\gamma_m$ = 19.6 kN/m$^3$ and $G_s$ = 2.72. Find the void ratio, degree of saturation and the dry unit weight.
*(Ans. 0.613, 82.0%, 16.54 kN/m$^3$)*

5. A one metre thick fill was compacted by a vibrating roller, and there was 30 mm reduction in the fill thickness. The initial void ratio was 0.94. What would be the void ratio after compaction?
*(Ans. 0.883)*

6. A 75 mm diameter and 20 mm thick cylindrical saturated clay specimen has a mass of 164.1 g. When dried in the oven at 105$^o$C, the mass is reduced to 121.3 g. What is the specific gravity of the soil grains?
*(Ans. 2.66)*

7. A section of canal 200 m long and 10 m wide is being deepened 1 m by means of a dredge. The effluent from the dredging operation is found to have a unit weight of 12.5 kN/m$^3$. The soil at the bottom of the canal has an in-place unit weight of 18.6 kN/m$^3$. The specific gravity of the soil grains is 2.69. If the effluent is to be pumped out at the rate of 500 litres/minute, how many operational hours would be required to complete the dredging work?

*(Ans. 218 hours)*

8. Soil for a compacted earth fill is available from three different borrow sites. At the earth fill, the soil is to be compacted to a void ratio of 0.62 with a finished volume of 150,000 $m^3$. The in situ void ratios and the costs (soil and transportation) per cubic metre for the three sites are given below. Which site would be economical?

| Borrow | Void ratio | Cost/$m^3$ |
|---|---|---|
| X | 0.85 | $7.80 |
| Y | 1.1 | $7.50 |
| Z | 1.4 | $6.60 |

*(Ans. Borrow A)*

9. A subbase for an airport runway, 100 m wide, 2000 m long and 500 mm thick, is to be constructed out of a clayey sand excavated from a nearby borrow, where the in situ water content is 6%. This soil is being transported in trucks having a capacity of 8 $m^3$, where each load weighs 13.2 tonnes. In the subbase course, the soil will be placed at a water content of 14.2% to a dry density of 1.89 t/$m^3$.
   (a) How many truck loads will be required to complete the job?
   (b) How many litres of water should be added to each truck load?
   (c) If the subbase becomes saturated later, without any change in volume, what would be the water content? Assume $G_s$ = 2.7.
   *(Ans. 15181, 1.03x$10^6$ litres, 17.3%)*

10. A soil to be used in the construction of an embankment is obtained by hydraulic dredging of a nearby canal. The embankment is to be placed at a dry density of 1.72 t/$m^3$ and will have a finished volume of of 20,000 $m^3$. The in place saturated density of the soil at the bottom of the canal is 1.64 t/$m^3$. The effluent from the dredging operation, having a density of 1.43 t/$m^3$, is pumped to the embankment site at a rate of 600 litres/minute. The specific gravity of the soil grains is 2.70.
   (a) How many operational hours would be required to dredge sufficient soil for the embankment?
   (b) What would be the volume of excavation at the bottom of the canal?
   *(Ans. 1396 hours, 90838 $m^3$)*