Chapter 7
Permeability and Seepage

7.1 INTRODUCTION

Permeability, as the name implies (ability to permeate), is a measure of how easily a fluid can flow through a porous medium. In geotechnical engineering, the porous medium is soils and the fluid is water at ambient temperature. Generally, coarser the soil grains, larger the voids and larger the permeability. Therefore, gravels are more permeable than silts. Hydraulic conductivity is another term used for permeability, often in environmental engineering literature.

Flow of water through soils is called *seepage*. Seepage takes place when there is difference in water levels on the two sides of the structure such as a dam or a sheet pile as shown in Fig. 1. Whenever there is seepage (e.g., beneath a concrete dam or a sheet pile), it is often necessary to estimate the quantity of the seepage, and permeability becomes the main parameter here.

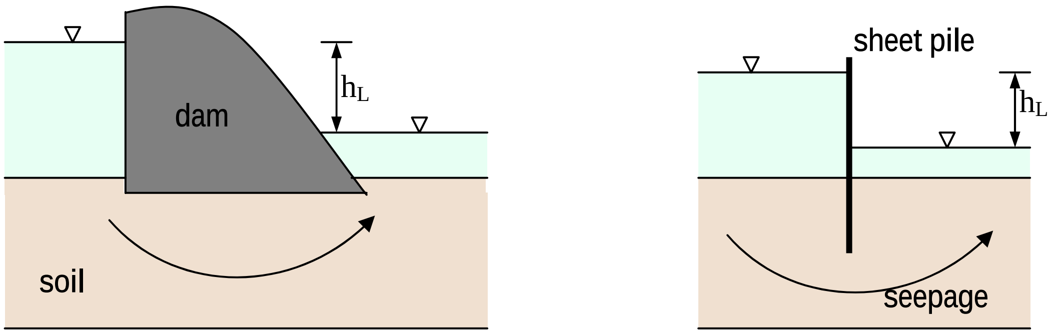


Figure 7.1 Seepage beneath (a) a concrete dam (b) a sheet pile

Sheet piles are interlocking walls, made of steel, timber or concrete segments. They are used water front structures and cofferdams (temporary structure made of interlocking sheet piles, making up an impermeable wall surrounding an area, often for construction works as in Fig. 2.)

Figure 2. Cofferdam at Montgomery Point Lock, USA (Courtesy: U.S.Army Corps of Engineers 2004)

7.2 BERNOULLI'S EQUATION

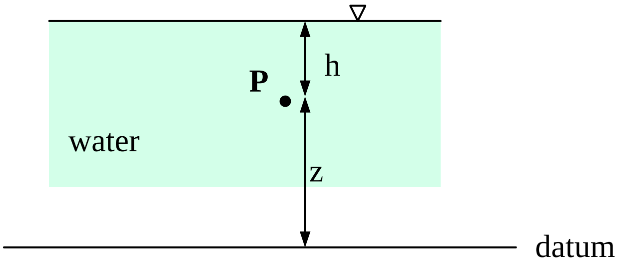


Figure 7.3. Total head at a point

Bernoulli's equation in fluid mechanics states that, for steady flow of non-viscous incompressible flow, the *total head* at a point can be expressed as the summation of three independent components, namely, *pressure head*, *elevation head* and *velocity head*. This is shown in Eq. 7.1.

$$\text{Total head} = \text{Pressure head} + \text{Elevation head} + \text{Velocity head}$$
$$= \frac{p}{\rho_w g} + z + \frac{v^2}{2g} \qquad (7.1)$$

where p is the pressure and v is the velocity at a point (P in Fig. 7.3) within the region of flow. The total head and three components in Eq. 7.1 have the units of length. The second component, elevation head, is measured with respect to an arbitrarily selected datum. It is simply the vertical distance above the horizontal datum line. If the point is below the datum, the elevation head is negative. At point P (Fig. 7.3), the pressure p in Eq. 7.1 is $h\rho_w g$, and therefore the pressure head is h.

7.3 FLOW THROUGH SOILS

When water flows through soils, whether beneath a concrete dam or a sheet pile, the seepage velocity is often very small. It is even smaller when squared, and the third component in Eq. 7.1 becomes negligible compared to the first two components. Therefore, Bernoulli's equation for flow through soils becomes:

$$\text{Total head} = \text{Pressure head} + \text{Elevation head}$$
$$= \frac{p}{\rho_w g} + z \qquad (7.2)$$

Neglect the velocity head in flow through soils.

When water flows through soils, from *upstream* to *downstream*, due to difference in water level as in Fig. 7.1, some energy is lost in overcoming the resistance provided by the soils. This loss of energy, expressed as total head loss (h_L), is simply the difference in water levels. The pressure p is the pore water pressure (u), and therefore pore water pressure at any point in the flow region can be written as:

$$u = \text{Pressure head} \times \rho_w g \qquad (7.3)$$

In Fig. 7.3, if h = 3 m, the pressure head and pore water pressure at P are 3 m and 29.43 kPa respectively.

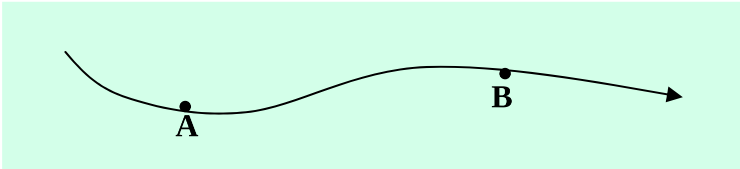


Figure 7.4 Hydraulic gradient

Hydraulic gradient is the total head loss per unit length. When water flows from point A to point B as shown in Fig. 7.4, the total head at A has to be greater than that at B. The average hydraulic gradient between A and B, is the total head lost between A and B divided by the length AB along the flow path.

$$i_{A-B} = \frac{\text{Total head at A - Total head at B}}{\text{length AB}} \qquad (7.4)$$

The hydraulic gradient is a constant in a homogeneous soil, since it is a measure of the head loss per unit length. It is dimensionless. If the soil is not homogeneous, the hydraulic gradient can vary from point to point.

EXAMPLE 7.1

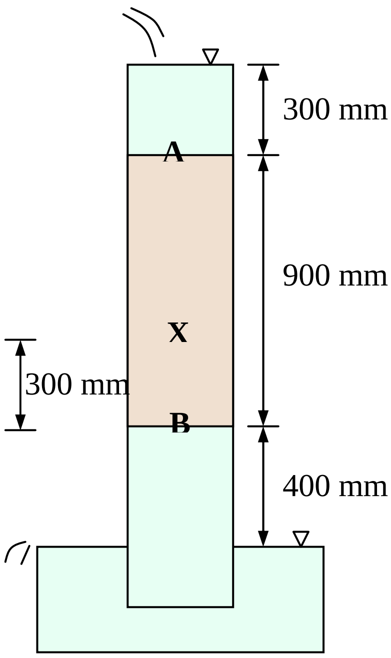


Figure 7.5

A 900 mm long cylindrical soil sample, contained as shown in Fig. 7.5, is subjected to a steady state flow under constant head. Find the pore water pressure at a point X.

<u>Solution:</u>

Let's take the tail-water level as the datum.

Total head loss across the specimen is 1600 mm.
$\therefore$ average hydraulic gradient within the soil = 1600/900 = 1.78

Total head at A = 1600 mm
For the flow from A to X,

$$\frac{\text{Total head at A - Total head at X}}{\text{length from A to X}} = 1.78$$

$$\frac{1600 - \text{Total head at X}}{600} = 1.78 \rightarrow \quad \text{Total head at X = 532.0 mm}$$

Elevation head at X = 700.0 mm
$\therefore$ Pressure head at X = -168.0 mm

$\therefore$ Pore water pressure at X = -(0.168)(9.81) = -1.648 kPa

> In seepage problems I generally select the *tail water* or *downstream water level* as the datum. The choice of datum can only affect the elevation and total head, but not the pressure head or the pore water pressure.

7.4 DARCY'S LAW

In 1856, a French engineer Darcy proposed that, what the flow through soils is laminar, the discharge velocity (v) is proportional to the hydraulic gradient (i). Darcy's law is thus:

$$v \propto i$$

$$v = k\,i \qquad (7.5)$$

Here, the constant *k* is known as the *coefficient of permeability* or simply *permeability*. It is also called *hydraulic conductivity*. Since *i* is dimensionless, *k* has the unit of velocity. In geotechnical engineering *k* is commonly expressed in cm/s (although m/s is the preferred metric unit), and other possible units include m/s, m/day, and mm/hour. In mining engineering, mm/hour is the preferred unit for permeability of mine fills and bricks.

In coarse grained soils, the effective grain size D_{10} has good correlation with permeability. Hazen (1911) suggested that, for uniform sands (C_u < 5) having D_{10} of 0.1-3 mm, in its loosest state, k and D_{10} are related by:

$$k\ (\text{cm/s}) = D_{10}^2\ (\text{cm}) \qquad (7.6)$$

There have been attempts to correlate permeability with e^2, $e^2/1+e$, $e^3/1+e$. One can intuitively see that larger the D_{10} or e, larger the void volume and thus larger the permeability.

Typical permeability values for the common soil types, and what these mean when it comes to drainage characteristics, are summarized in Table 7.1 (Terzaghi et al. 1996). When k is less that 10^{-6} cm/s, the soil is practically impervious.

Table 7.1. Permeability and drainage characteristics of soils (Terzaghi et al. 1996)

	Permeability (m/s)											
	10^{0}	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}	10^{-8}	10^{-9}	10^{-10}	10^{-11}
Drainage	Good						Poor		Practically impervious			
Soil Types	Clean gravel		Clean sands, clean sand & gravel mixtures			Very fine sands, organic & inorganic silts, mixtures of sand silt & clay, glacial till, stratified clay deposits, etc.				Impervious soils e.g., homogeneous clays below zone of weathering		
					"Impervious" soils modified by effects of vegetation & weathering							

It is good to have an idea about the *order of magnitude* for the permeability of a specific soil type.

7.5 LABORATORY DETERMINATION OF PERMEABILITY

Permeability of a coarse grained soil can be determined by a *constant head permeability test* (AS1289.6.7.1-2001; ASTM D2434), and in a fine grained soil, *falling head permeability test* (AS1289.6.7.2-2001; ASTM D5856) works the best. In a constant head permeability test (Fig. 7.6), the total head loss (h_L) across a cylindrical soil specimen of length L and cross sectional area A, is maintained constant throughout the test, and at steady state, the flow rate (Q) is measured.

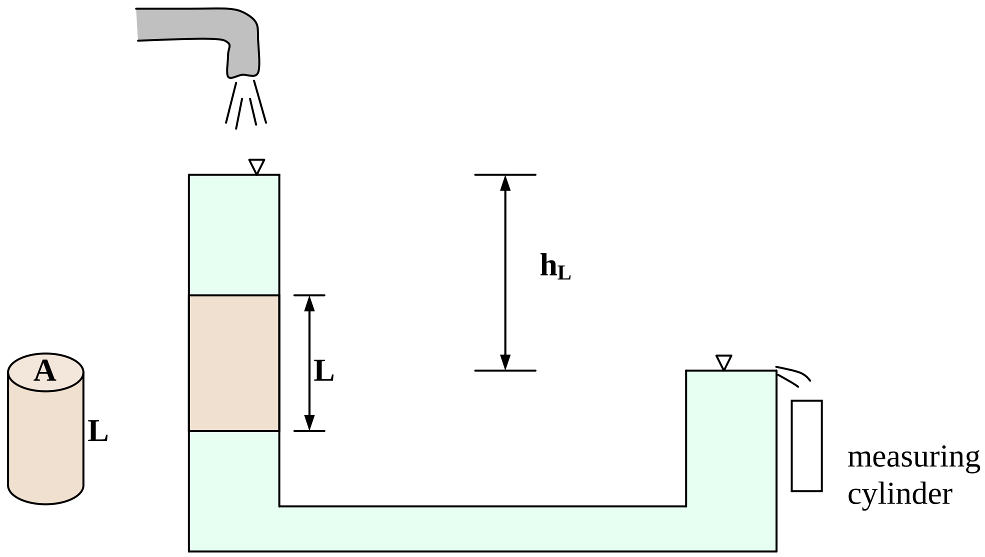


Figure 7.6 Constant head permeability test

Therefore, the discharge velocity (v) is given by:

$$v = Q/A$$

The hydraulic gradient (i) across the soil specimen is h_L/L. Applying Darcy's law,

$$Q/A = k\, h_L/L$$

Therefore, k is given by:

$$k = \frac{QL}{h_L A} \qquad (7.7)$$

Why can't we do constant head permeability test on fine grained soils? It just takes quite a long time to collect a measurable quantity of water to compute the flow rate. A simplified schematic diagram for a falling head permeability setup is shown in Fig. 7.7. The cylindrical soil specimen has cross sectional area of A and length L. The standpipe has internal cross sectional area of a.

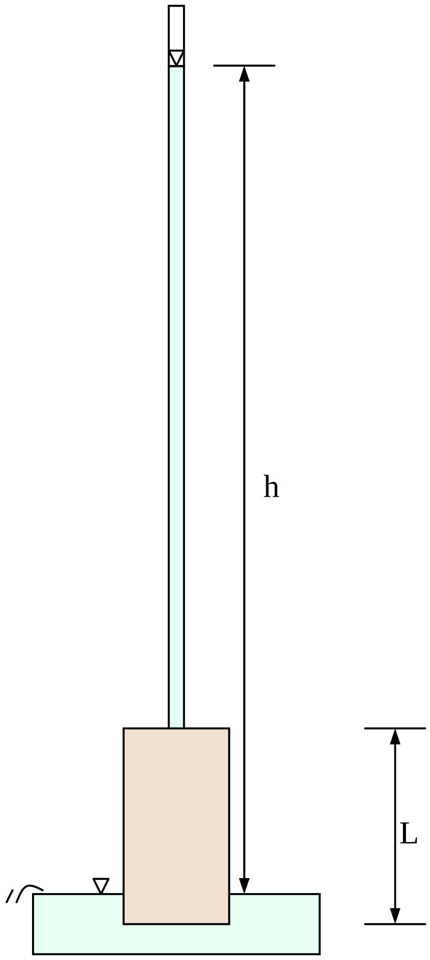


Figure 7.7 Schematic diagram of a falling head permeability test setup

By applying Darcy's law, and equating the flow rate in the standpipe and the soil specimen, it can be shown that the permeability can be computed from Eq. 7.8

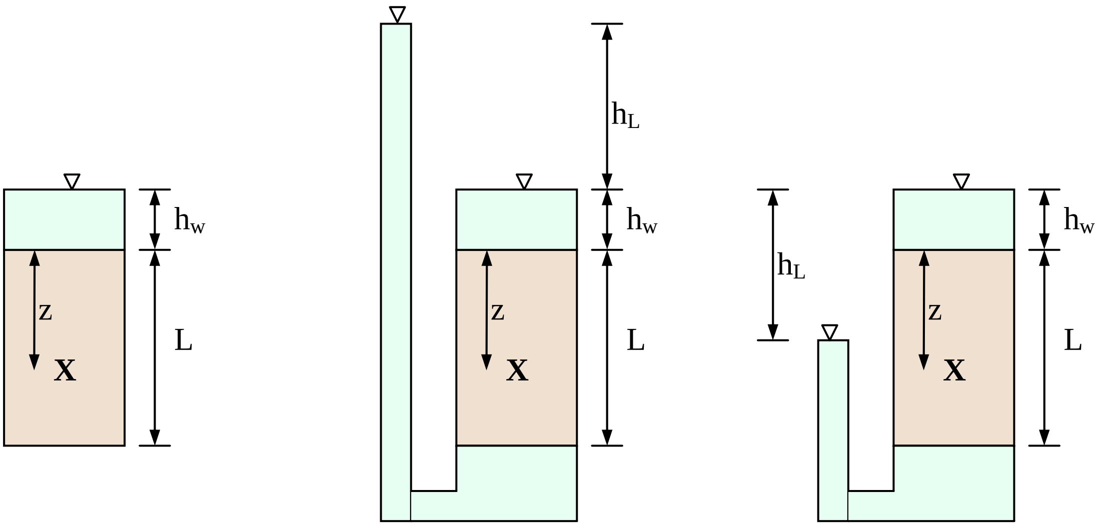
$$k = \frac{aL}{At}\ln\left(\frac{h_1}{h_2}\right) \qquad (7.8)$$

Here, *t* is the time taken for the water level in the standpipe to fall from h_1 to h_2.

Why can't we do falling head permeability test on coarse grained soils? The flow rate is so high that water level will drop from h_1 to h_2 within a few seconds, not giving us enough time to take the measurements properly.

Permeability in the field can be measured through a "pump-in" or "pump-out" test on a well or bore hole. Here, the flow rate to maintain the water table at a specific height is measured and the permeability can be computed using some analytical expressions found in textbooks.

7.6 STRESSES IN SOILS DUE TO FLOW



(a) (b) (c)

Figure 7.8 Three different scenarios (a) Static (b) Flow-up (c) Flow-down

Three different scenarios, of identical soil specimens subjected to different flow conditions, are shown in Fig. 7.8. In Fig. 7.8 a, there is no flow and the water is *static*. In Figs. 7.8 b, flow takes place due to a head difference of h_L across the specimen, and the flow is *upwards* through the specimen. In Fig. 7.8 c, the flow through the specimen is *downwards*, again due to a head difference of h_L. When there is flow, the hydraulic gradient i is given by h_L/L.

For all three situations, the total vertical stress is the same. The pore water pressures and effective stresses are summarized below.

(a) Static situation:
$\sigma_v = \gamma_w h_w + \gamma_{sat} z$
$u = \gamma_w (h_w + z)$
$\sigma_v' = \gamma' z$

(b) Flow-Up Situation:
$\sigma_v = \gamma_w h_w + \gamma_{sat} z$
$u = \gamma_w (h_w + z) + \mathbf{i\,z\,\gamma_w}$
$\sigma_v' = \gamma' z - \mathbf{i\,z\,\gamma_w}$

(c) Flow-Down Situation:
$\sigma_v = \gamma_w h_w + \gamma_{sat} z$
$u = \gamma_w (h_w + z) - \mathbf{i\,z\,\gamma_w}$
$\sigma_v' = \gamma' z + \mathbf{i\,z\,\gamma_w}$

When the flow is upwards in the soil, pore water pressure increases and effective stress decreases. When the flow is downward, the pore water pressure decreases and the effective stress increases. Higher the hydraulic gradient, higher the increase or decrease in the values of pore pressure and effective stress.

Now let's have a closer look at the flow-up situation, in a *granular soil*. The effective stress is positive as long as $\gamma' z$ is greater than $iz\gamma_w$. If the hydraulic gradient is too large, $iz\gamma_w$ can exceed $\gamma' z$, and the effective vertical stress can become negative. This implies that there is no inter-particle contact stress, and the grains are no longer in contact. When this occurs, the granular soil is said to have reached *quick condition*. The hydraulic gradient at this situation is known as the *critical hydraulic gradient (i_c)*, which is given by Eq. 7.9.

$$i_c = \frac{\gamma'}{\gamma_w} = \frac{G_s - 1}{1 + e} \qquad (7.9)$$

The same thing causes quick sand you may have seen in movies, and liquefaction of granular soils subjected to vibratory loadings. Here, sudden rise in pore water pressures can bring the effective stresses to almost zero.

7.7 SEEPAGE

Let's have a look at the concrete dam and sheet pile in Fig. 7.1, where seepage takes place through the sub soil, due to head difference between up stream and down stream water levels. If we know the permeability of the soil, how do we compute the discharge through the soil? How do we compute the pore water pressures at various locations in the flow region or assess the uplift loading on the bottom of the concrete dam? Is there any problem with hydraulic gradient being too high within the soil? To address all these, let's look at some fundamentals in flow.

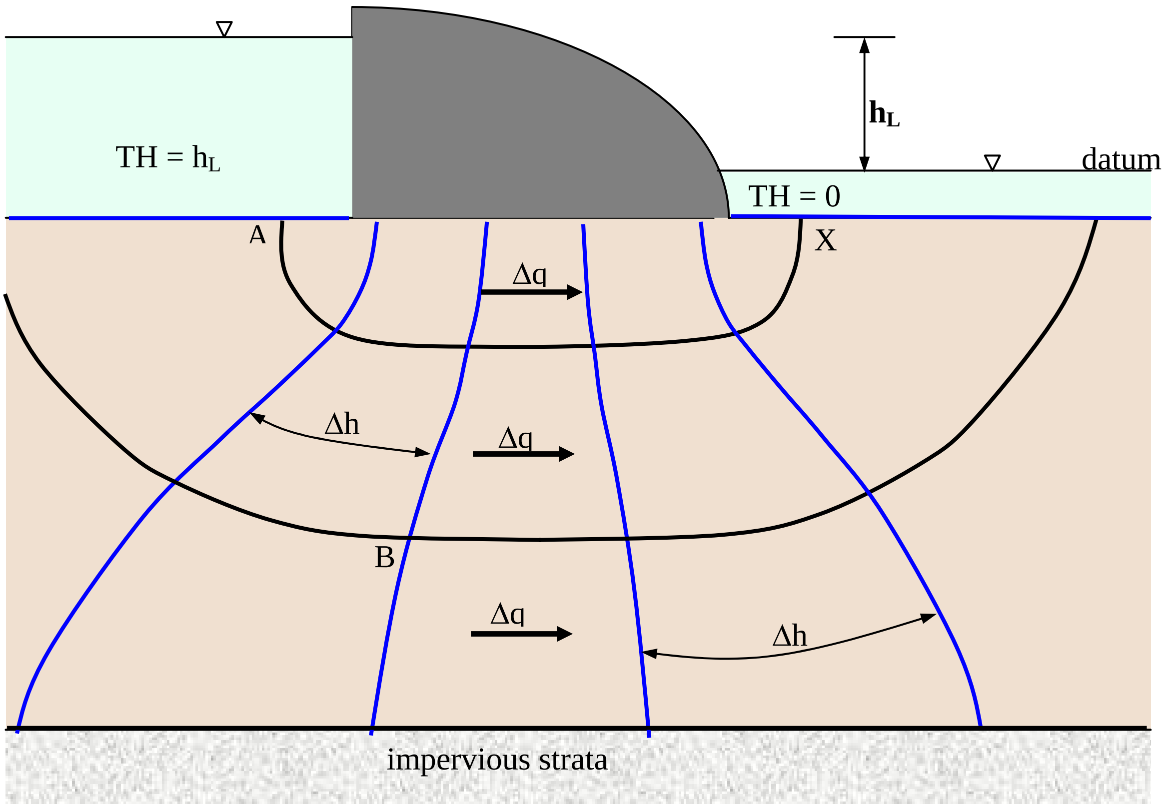


Figure 7.9 Streamlines and equipotential lines

In the flow beneath the concrete dam shown in Fig. 7.9, we will assume that the concrete dam is impervious, and an impervious stratum such as bedrock or stiff clay underlies the soil.

Let's select the datum as the downstream water level. That makes the total head within the downstream and upstream water 0 and h_L respectively. There is total head *loss* of h_L along each streamline originating upstream and ending up downstream. This loss in head or energy is used against overcoming the resistance to flow provided by the soil. Here, streamline or flow line, is the path of a water molecule in the flow region. There are thousands of streamlines in the flow region. The flow passage between two adjacent streamlines is known as *flow channel*. In a flow net, we only draw streamlines such that the discharge is the same (= Δq) through each of the flow channels. Let's say there are N_f flow channels in the flow net.

An *equipotential line* is a contour of constant total head. The blue lines shown in the figure are all equipotential lines, where the total head is constant along each of them. In a *flow net*, such as the one shown in Fig. 7.9, the equipotential lines are drawn such that the total head difference between two adjacent ones is the same (= Δh) throughout the flow region. If there are N_d equipotential drops in a flow net, $\Delta h = h_L/N_d$. In Fig. 7.9, $\Delta h = 0.2\ h_L$.

N_f and N_d do not have to be integers. In Fig. 7.9, they are 3 and 5 respectively. The discharge through the soil, per unit thickness (perpendicular to the paper), can be given by:

$$q = kh_L \frac{N_f}{N_d} \qquad (7.10)$$

(Note: The flow net I have drawn above is quite poor. You can see much better quality flow nets in textbooks.)

From the flow net, the total head at any point can be computed. For example, at point B in Fig. 7.9, total head is $h_L - 2\ \Delta h$ (= $3\ \Delta h$).

EXAMPLE 7.2

Flow takes place through a 100 mm diameter and 275 mm long soil sample, from top to bottom, as shown in the figure below. The manometers are 120 mm apart, and the water level difference within the two manometers is 100 mm at steady state. If the permeability of the soil is 3.7×10^{-4} cm/s, what is the flow rate?

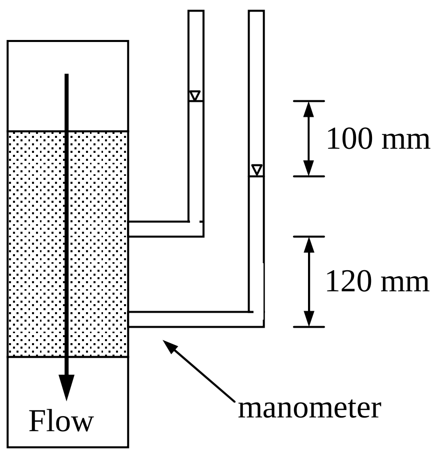


Fig. 7.10 Example 7.2

Solution:

Hydraulic gradient across the soil specimen = 100/120 = 0.833

∴ velocity of flow = k i =(3.7 x 10^{-4})(0.833) = 3.082 x 10^{-4} cm/s
Cross sectional area of the specimen = 78.54 cm^2
∴ flow rate = (3.082 x 10^{-4})(78.54) = 0.0242 cm^3/s = 1.45 cm^3/min

EXAMPLE 7.3

A long horizontal drain at 3 m depth collects the ground water in a low-lying area. The free water table coincides with the ground level and the flownet for the ground water flow is shown in Fig. 1. The 6 m thick sandy clay bed is underlain by an impervious stratum. Permeability of the sandy clay is 6.2×10^{-5} cm/s.

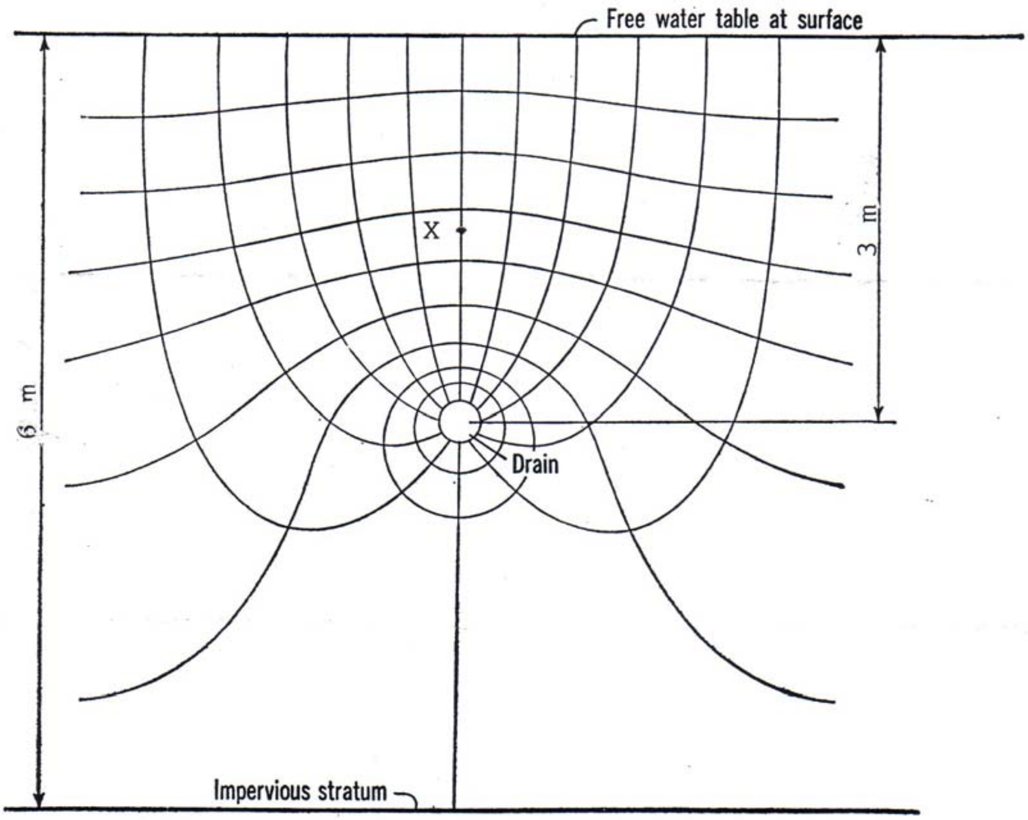


Figure 7.11. Flownet around the Horizontal Drain for Ex. 7.3

(a) Find the discharge through the drain in m^3/day, per metre length of drain.
(b) Find the pore water pressure at X, 1.5 m into the soil, directly above the drain.
(c) Estimate the velocity of flow at X.

Solution:
Let's take datum at the drain level. This makes the total head at the top of the ground (and water table at the surface) 3.0 m. Head loss from the surface to drain = 3.0 m.
Let's assume water is at atmospheric pressure in the drain and at the surface (ground level).

(a) Let's consider only half of the flow region.
h_L = 3.0 m; N_f = 6; and N_d = 9

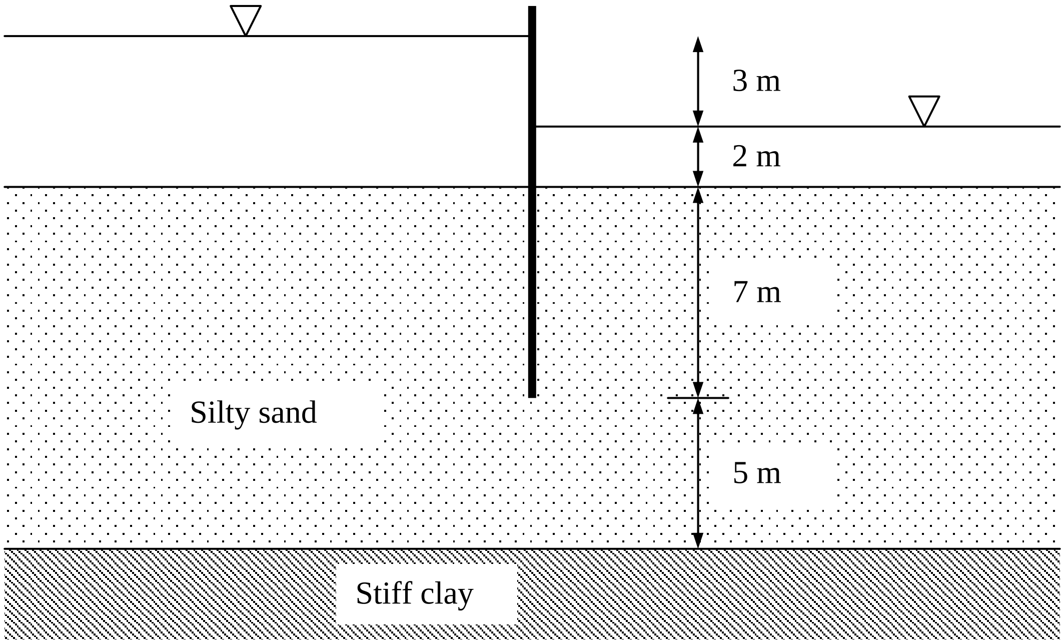
$$Q = kh_L \frac{N_f}{N_d}$$

$$= (6.2\times10^{-7}\, m/s)(3m)\left(\frac{2\times6}{9}\right)(24\times3600) = 0.214\ \mathrm{m}^3/day \text{ per m}$$

(b) Change in total head between two equipotential drops $\Delta h = 3/9 = 0.333$ m.
$\therefore$ Total head at X = 3.0 – 3.4 Δh = 1.867 m
Elevation head at X = 1.5 m
$\therefore$ Pressure head at X = 1.867 – 1.500 = 0.367 m
$\therefore$ Pore water pressure at X = (0.367)(9.81) = 3.60 kPa

(c) The vertical length of the curvilinear square at X is measured as 0.4 m
$\therefore$ Hydraulic gradient at X = 0.333/0.4 = 0.83
$\therefore$ velocity = k i = $6.2 \times 10^{-5} \times 0.83 = 5.15 \times 10^{-5}$ cm/s.

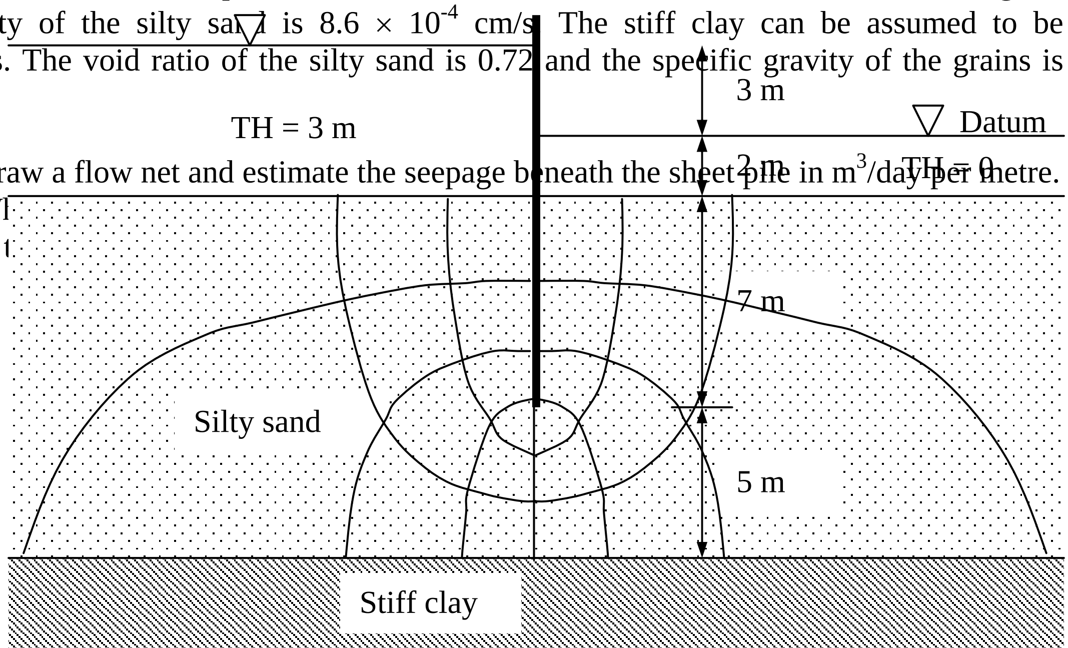
EXAMPLE 7.4



A stiff clay layer underlies a 12 m thick silty sand deposit. A sheet pile is driven into the sand to a depth of 7 m, and the upstream and downstream water levels are as shown in the figure. Permeability of the silty sand is 8.6×10^{-4} cm/s. The stiff clay can be assumed to be impervious. The void ratio of the silty sand is 0.72 and the specific gravity of the grains is 2.65.

(a) Draw a flow net and estimate the seepage beneath the sheet pile in m^3/day per metre.
(b) Wh
(c) Is t

Solution:



The flow net I have drawn here is very ordinary. You should be able to draw a better one using pencil.

(a) In the flow net, $N_f = 3$; $N_d = 8$; $h_L = 3$ m. The flow (Q) is given by:

$$Q = kh_L \frac{N_f}{N_d} = (8.6\times10^{-6})(3)\left(\frac{3}{8}\right)(24\times3600) = 0.836\,\text{m}^3/day \text{ per metre}$$

(b) Taking downstream water level as the datum, at the tip of the sheet pile,

Total head = 1.5 m

Elevation head = -9 m

$\therefore$ Pressure head = 1.5 – (-9) = 10.5 m

Pore water pressure = (10.5)(9.81) = 103.0 kPa

(c) Head loss per equipotential drop, Δh = 3/8 = 0.375 m

The maximum exit hydraulic gradient (near the sheet pile) = 0.375/2.6 = 0.144

The critical hydraulic gradient (i_c) is given by:

$$i_c = \frac{G_s - 1}{1+e} = \frac{2.65-1}{1+0.72} = 0.96$$

$\therefore$ Safety factor with respect to piping F = 0.96/0.144 = 6.7 > 5

The arrangement is quite safe with respect to piping.

REFERENCES

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