

Comparative Analyses of Effective Stiffness Matrix Results obtained by Three Different Methods for Transversal Isotropic Geological Material

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ABSTRACT: The determination of effective stiffness matrix is very important for civil engineering researchers, the first key factor is to calculate the effective stiffness matrix correctly in order to obtain the rational results. Two sorts method of current effective stiffness matrix determination are volumetrical average method and self-consist method for every element in the stiffness matrix. Based on the homogenization technology, the formula of macroscopic effective stiffness matrix is obtained for transversal isotropic geological material under the condition that the displacement and stress are equal in the everywhere in the element, the proposed equation can get the formulas of volumetrical average method and self – consist method under some special cases. The comparative analyses of three different methods are performed; the calculating results show the proposed method can describe the parallel flow model and series flow model. The foundational results obtained in this paper are very important for calculation and analyses of rock slide and dam foundation engineering.

1 INTRODUCTION

The stiffness matrix calculation is very important for numerical analyses of the geological material, lots of researches (Dragon A., 1993; J.F. Shao, 1998; J.F. Shao, 2000; Kemeny J.M., 1991; Krajcinovic D., 1981; Y.F. Lu, 2002; LU Yingfa, 2005; James G., 1986; J.-L. Auriault, 1990; Kachanov M., 1992; Kachanov M., 1993) have been studied the theories of stiffness matrix determination of the geological media, some calculating methods have been proposed. For instance: 1) Statistical method^[1]: the macroscopic stiffness matrix are obtained for the geological media, based on the statistical analyses of joints and fissures in the geological media. 2) Homogenization technology (J.-L. Auriault, 1990): the macroscopic effective stiffness is obtained by taking the average by the volume for component medium in RVE (representative volume element), plenty of the researchers accept this method in the theory and engineering application; 3) Self-consist method (James G., 1986): the macroscopic stiffness is obtained by volume calculation for the different component of geological media, 4) Differential scheme (James G., 1986): the variable law between macroscopic stiffness and microscopic stiffness is described by differential equation. The statistical method, homogenization technology and self-consist method are fit for the arbitrary distribution of microscopic cracks, microscopic voids and microscopic joints in the porous media; the homogenization method is not fit for the arbitrary distribution of microscopic cracks, microscopic voids and microscopic joints, but fit for the periodic distribution of microscopic cracks, microscopic voids and microscopic joints in the porous media. Differential scheme is not fit for the concentration distribution of micro-crack, but fit for the arbitrary distribution of micro-cracks in the porous media. Kachanov (Kachanov M., 1992; Kachanov M., 1993) proposed the macroscopic effective stiffness can be obtained by GIBBS free energy, the GIBBS function is related not to stress tensor, but to density of micro-cracks in porous media. In this paper, the macroscopic stiffness matrix is analyzed and investigated for layered geological material, the

macroscopic stiffness matrix is obtained based on the microscopic analyses, and the obtained results are compared with the other methods.

2 HOMOGENIZATION TECHNOLOGY

Generally, X: presents the macroscopic variable, the correspondent coordinate is (x_1, x_2, x_3) , Y: presents the microscopic variable, the correspondent coordinate is (y_1, y_2, y_3) for the described geological materials. L: presents the macroscopic scale, l: presents the microscopic scale, let: $e=l/L$, of course, $e<1.0$. The arbitrary function is related with the macroscopic variable X and the microscopic variable Y. the displacement function can be described with the macroscopic coordinate and microscopic coordinate in the following form for the RVE:

$$u_i^m(x, y) = u_i^{m(0)}(x, y) + e^1 u_i^{m(1)}(x, y) + e^2 u_i^{m(2)}(x, y) + \dots i = 1, 2, 3 \quad (1)$$

(m: the numbers of microscopic medium)

The strain for the different microscopic medium:

$$\varepsilon_{ij}^m = e^{-1} \frac{1}{L} \frac{\partial u_i^{m(0)}}{\partial y_j} + e^0 \frac{1}{L} \left[\frac{\partial u_i^{m(0)}}{\partial x_j} + \frac{\partial u_i^{m(1)}}{\partial y_j} \right] + e \frac{1}{L} \left[\frac{\partial u_i^{m(1)}}{\partial x_j} + \frac{\partial u_i^{m(2)}}{\partial y_j} \right] + \dots \quad (2)$$

The stress can be obtained under the condition that the Hook Law can describe the elastic mechanical behaviours for the different microscopic medium:

$$\sigma_{ij}^m = \frac{1}{L} \{ e^{-1} C_{ijkh}^m \frac{\partial u_k^{m(0)}}{\partial y_h} + e^0 C_{ijkh}^m \left[\frac{\partial u_k^{m(0)}}{\partial x_h} + \frac{\partial u_k^{m(1)}}{\partial y_h} \right] + e C_{ijkh}^m \left[\frac{\partial u_k^{m(1)}}{\partial x_h} + \frac{\partial u_k^{m(2)}}{\partial y_h} \right] + e^2 C_{ijkh}^m \left[\frac{\partial u_k^{m(2)}}{\partial x_h} + \frac{\partial u_k^{m(3)}}{\partial y_h} \right] + \dots \} \quad (3)$$

The balance equation is also obtained, when the specific weight is neglected:

$$\begin{aligned} \frac{\partial \sigma_{ij}^m}{\partial x_j} = & \frac{1}{L^2} \{ e^{-2} C_{ijkh}^m \frac{\partial^2 u_k^{m(0)}}{\partial y_h \partial y_j} + e^{-1} C_{ijkh}^m \left[\frac{\partial^2 u_k^{m(0)}}{\partial y_h \partial y_j} + \frac{\partial}{\partial y_j} \left(\frac{\partial u_k^{m(0)}}{\partial x_h} + \frac{\partial u_k^{m(1)}}{\partial x_h} \right) \right] + \\ & e^0 C_{ijkh}^m \left[\frac{\partial}{\partial x_j} \left(\frac{\partial u_k^{m(0)}}{\partial x_h} + \frac{\partial u_k^{m(1)}}{\partial y_h} \right) + \frac{\partial}{\partial y_j} \left(\frac{\partial u_k^{m(1)}}{\partial x_h} + \frac{\partial u_k^{m(2)}}{\partial y_h} \right) \right] + \\ & e^1 C_{ijkh}^m \left[\frac{\partial}{\partial x_j} \left(\frac{\partial u_k^{m(1)}}{\partial x_h} + \frac{\partial u_k^{m(2)}}{\partial y_h} \right) + \frac{\partial}{\partial y_j} \left(\frac{\partial u_k^{m(2)}}{\partial x_h} + \frac{\partial u_k^{m(3)}}{\partial y_h} \right) \right] + \dots \} = 0 \end{aligned} \quad (4)$$

(m: is not summed)

Let the coefficient of e^n is equal to zero in the balance equation, the following equation are presented:

$$e^{-2} : C_{ijkh}^m \frac{\partial^2 u_k^{m(0)}}{\partial y_j \partial y_h} = 0 \quad (5-1)$$

$$e^{-1} : C_{ijkh}^m \frac{\partial^2 u_k^{m(0)}}{\partial y_h \partial x_j} + C_{ijkh}^m \frac{\partial}{\partial y_j} \left[\frac{\partial u_k^{m(0)}}{\partial x_h} + \frac{\partial u_k^{m(1)}}{\partial y_h} \right] = 0 \quad (5-2)$$

$$e^0 : C_{ijkh}^m \frac{\partial}{\partial x_j} \left[\frac{\partial u_k^{m(0)}}{\partial x_h} + \frac{\partial u_k^{m(1)}}{\partial y_h} \right] + C_{ijkh}^m \left[\frac{\partial^2 u_k^{m(1)}}{\partial y_j \partial x_h} + \frac{\partial^2 u_k^{m(2)}}{\partial y_j \partial y_h} \right] = 0 \quad (5-3)$$

$$e^1 : C_{ijkh}^m \frac{\partial}{\partial x_j} \left[\frac{\partial u_k^{m(1)}}{\partial x_h} + \frac{\partial u_k^{m(2)}}{\partial y_h} \right] + C_{ijkh}^m \left[\frac{\partial^2 u_k^{m(2)}}{\partial y_j \partial x_h} + \frac{\partial^2 u_k^{m(3)}}{\partial y_j \partial y_h} \right] = 0 \quad (5-4)$$

Let the coefficient of displacement e , of strain and stress e^0 and of the balance equation e^1 is equal to zero respectively, the results can be obtained from Eq.(5) in the following form:

$$u_i^{m(0)} = a_i(x), \quad u_i^{m(1)} = u_i^{m(1)}(y) \quad (6)$$

$$\varepsilon_{ij}^m = \frac{\partial u_i^{m(0)}}{\partial x_j} + \frac{\partial u_i^{m(1)}}{\partial y_j} = E_{ij}^0 + e_{ij}^{*m} \quad (7)$$

$$\Sigma_{ij}^m = C_{ijkh}^m (E_{kh}^0 + \varepsilon_{kh}^{*m}) \quad (8)$$

$$C_{ijkh}^m \frac{\partial}{\partial y_j} \left[\frac{\partial u_k^{m(0)}}{\partial x_h} + \frac{\partial u_k^{m(1)}}{\partial y_h} \right] = 0 \quad (9)$$

Where: $\frac{\partial u_i^{m(0)}}{\partial x_j} = \frac{\partial a_i(x)}{\partial x_j} = E_{ij}^0$; $\frac{\partial u_i^{m(1)}}{\partial y_j} = e_{ij}^{*m}$.

The two different media are discussed here, if the following relationships are supposed:
The microscopic strain is satisfied the following equation:

$$n_1 \varepsilon_{kh}^{*1} + n_2 \varepsilon_{kh}^{*2} = 0 \quad (10)$$

The sum of micro-force for the different medium is equal to zero:

$$n_1 C_{ijkh}^1 \varepsilon_{kh}^{*1} + n_2 C_{ijkh}^2 \varepsilon_{kh}^{*2} = 0 \quad (11)$$

The stress is equal for different microscopic porous medium everywhere:

$$(C_{ijkh}^1 - C_{ijkh}^2) E_{kh}^0 = C_{ijkh}^2 \varepsilon_{kh}^{*2} - C_{ijkh}^1 \varepsilon_{kh}^{*1} \quad (12)$$

When the macroscopic stress is satisfied the volumetrical distribution for the two different sort media, the effective macroscopic stiffness matrix (C^{eff}) can be obtained by virtue of the Eq.(10~12).

$$\frac{1}{C_{ijkh}^{eff}} = \frac{n_1}{C_{ijkh}^1} + \frac{n_2}{C_{ijkh}^2} = \left\langle \frac{1}{C_{ijkh}^m} \right\rangle \quad (13)$$

According to the process of deduction of Eq.(13), when the microscopic strain is neglected, the effective stiffness matrix is described by simple average method:

$$C_{ijkh}^{eff} = \left\langle C_{ijkh}^m \right\rangle \quad (14)$$

Where: n_1, n_2 present the volume ratio for two different media respectively.

3 RESULTS OF TRANSVERSAL ISOTROPIC STIFFNESS MATRIX OF LAYERED MEDIA

The media are composed of the two different isotropic geological materials (see Fig.1), the ratio of medium 1 is n , and the other ratio is $1-n$, the displacement function of two different materials is described in the following form:

$$u_i^m(x, y) = u_i^{(0)}(x) + e^1 u_i^{m(1)}(y) \quad i = 1, 2, 3 \quad m = 1, 2 \quad (15)$$

The displacement must be equal on the boundary of RVE:

$$u_i^1(x, y) \Big|_{y_i=0} = u_i^2(x, y) \Big|_{y_i=1} \quad i = 1, 2 \quad m = 1, 2 \quad (16)$$

The concrete formula is in the following form from Eq.(6~10):

$$u_i^m(x, y) = u_i^{(0)}(x) + e^1 (a_{i3} y_3 + b_i) \quad i = 1, 2, 3 \quad m = 1, 2 \quad (17)$$

Where: a_{i3} , b_i are constant.

The concord condition in y_3 condition can be described:

$$u_3^1(x, y) \Big|_{y_i=n} = u_3^2(x, y) \Big|_{y_i=n} \quad (18)$$

$$u_i^1(x, y) \Big|_{y_i=0} = u_i^2(x, y) \Big|_{y_i=1} \quad i = 1, 2, 3 \quad m = 1, 2 \quad (19)$$

The macroscopic stresses are balanced:

$$\Sigma_{ij}^1 \Big|_{y_3=n} = \Sigma_{ij}^2 \Big|_{y_3=n} \quad (20)$$

$$\Sigma_{ij}^1 \Big|_{y_3=0} = \Sigma_{ij}^2 \Big|_{y_3=1} \quad (21)$$

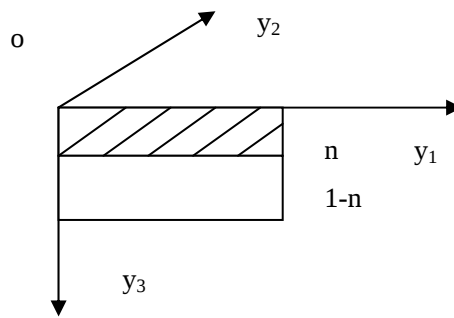


Fig.1 Representative scheme of transversal isotropic material

The effective stiffness matrix is obtained in the Eq.(22) from the Eq.(16~21).

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{12} \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} C_{1111}^{eff} & C_{1122}^{eff} & C_{1133}^{eff} & 0 & 0 & 0 \\ C_{2211}^{eff} & C_{2222}^{eff} & C_{2233}^{eff} & 0 & 0 & 0 \\ C_{3311}^{eff} & C_{3322}^{eff} & C_{3333}^{eff} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{2323}^{eff} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{1313}^{eff} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{1212}^{eff} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11}^0 \\ \varepsilon_{22}^0 \\ \varepsilon_{33}^0 \\ \varepsilon_{23}^0 \\ \varepsilon_{13}^0 \\ \varepsilon_{12}^0 \end{Bmatrix} \quad (22)$$

Where:

$$\begin{aligned} C_{1111}^{eff} &= n \cdot C_{1111}^1 + (1-n)C_{1111}^2 - n(1-n) \frac{(C_{1133}^2 - C_{1133}^1)(C_{3311}^2 - C_{3311}^1)}{(1-n)C_{3333}^1 + n \cdot C_{3333}^2} \\ C_{1122}^{eff} &= n \cdot C_{1122}^1 + (1-n)C_{1122}^2 - n(1-n) \frac{(C_{1133}^2 - C_{1133}^1)(C_{3322}^2 - C_{3322}^1)}{(1-n)C_{3333}^1 + n \cdot C_{3333}^2} \\ C_{1133}^{eff} &= n \cdot C_{1133}^1 + (1-n)C_{1133}^2 - n(1-n) \frac{(C_{1133}^2 - C_{1133}^1)(C_{3333}^2 - C_{3333}^1)}{(1-n)C_{3333}^1 + n \cdot C_{3333}^2} \\ C_{2222}^{eff} &= n \cdot C_{2222}^1 + (1-n)C_{2222}^2 - n(1-n) \frac{(C_{2233}^2 - C_{2233}^1)(C_{3322}^2 - C_{3322}^1)}{(1-n)C_{3333}^1 + n \cdot C_{3333}^2} \\ C_{2233}^{eff} &= n \cdot C_{2233}^1 + (1-n)C_{2233}^2 - n(1-n) \frac{(C_{2233}^2 - C_{2233}^1)(C_{3333}^2 - C_{3333}^1)}{(1-n)C_{3333}^1 + n \cdot C_{3333}^2} \\ C_{3333}^{eff} &= \frac{C_{3333}^1 C_{3333}^2}{(1-n)C_{3333}^1 + n \cdot C_{3333}^2}, \quad C_{2323}^{eff} = \frac{C_{2323}^1 \cdot C_{2323}^2}{(1-n)C_{2323}^1 + n \cdot C_{2323}^2} \\ C_{1313}^{eff} &= \frac{C_{1313}^1 \cdot C_{1313}^2}{(1-n)C_{1313}^1 + n \cdot C_{1313}^2}, \quad C_{1212}^{eff} = n \cdot C_{1212}^1 + (1-n)C_{1212}^2 \end{aligned} \quad (23)$$

4 ANALYSES

The five independent parameters are existed by virtue of variable principal of elastic mechanics for transversal isotropic material, the five independent parameters are: $C_{1111}^{eff}=C_{2222}^{eff}$, C_{1122}^{eff} , $C_{1133}^{eff}=C_{2233}^{eff}$, C_{3333}^{eff} , $C_{2323}^{eff}=C_{1313}^{eff}$. The self-consist method can be obtained (see Eq.(13)) in the section 3, when the sum of micro-stress and micro-strain is equal to zero respectively and the stress is equal everywhere in RVE. The macroscopic effective stiffness matrix can be described by the form of the sum of volume average for the different component material (see Eq. (14)), when the microscopic strain and microscopic stress are neglected for the RVE. However, when the limited conditions of displacement and force are satisfied in the RVE for transversal isotropic porous media, the effective stiffness matrix is gotten (see Eq.(23)). C_{1212}^{eff} satisfies the simple ratio rule, $C_{1313}^{eff}, C_{2323}^{eff}, C_{3333}^{eff}$ satisfies the self-consist rule, the others are not fit self - consist rule. The comparative analyses are performed between the two conventional methods (simple volume average method (method 1), self-consist method (method 2)) and the method proposed in this paper (method 3)). Two different media are discussed, medium 1: elastic modulus $E_1 = 35GPa$, Poisson: $\nu_1 = 0.15$; medium 2: elastic modulus $E_2 = 20GPa$, Poisson: $\nu_2 = 0.26$, the stiffness in the Eq.(22) can be presented:

$$\begin{aligned} C_{1111}^1 &= C_{2222}^1 = C_{3333}^1 = 36.96GPa, \quad C_{1122}^1 = C_{1133}^1 = C_{2233}^1 = 6.52GPa, \\ C_{1212}^1 &= C_{2323}^1 = C_{3131}^1 = 30.43GPa, \quad C_{1111}^2 = C_{2222}^2 = C_{3333}^2 = 24.47GPa, \\ C_{1122}^2 &= C_{1133}^2 = C_{2233}^2 = 8.60GPa, \quad C_{1212}^2 = C_{2323}^2 = C_{3131}^2 = 15.87GPa. \end{aligned}$$

The correspondent effective in the Eq. (23) are shown in the following:

$$\begin{aligned}
C_{1111}^{eff} &= 37n + 24.47(1-n) - \frac{4.31n(1-n)}{37(1-n) + 24.47n} = C_{2222}^{eff} \\
C_{1122}^{eff} &= 6.52n + 8.60(1-n) - \frac{4.31n(1-n)}{37(1-n) + 24.47n} \\
C_{1133}^{eff} &= 6.52n + 8.60(1-n) + \frac{25.92n(1-n)}{37(1-n) + 24.47n} = C_{2233}^{eff} \\
C_{3333}^{eff} &= \frac{905.4}{37(1-n) + 24.47n}, C_{2323}^{eff} = \frac{482.92}{30.43(1-n) + 15.87n} = C_{1313}^{eff} \\
C_{1212}^{eff} &= 30.43n + 15.87(1-n)
\end{aligned} \quad (24)$$

The formula of self-consistent method for the five independent elements (C_{1111}^{eff} , C_{1122}^{eff} , C_{1133}^{eff} , C_{3333}^{eff} , C_{2323}^{eff}) in the Eq.(22) are presented in the following:

$$\begin{aligned}
C_{1111}^{eff} &= \frac{905.4}{37(1-n) + 24.47n}, C_{1122}^{eff} = \frac{56.07}{6.52(1-n) + 8.60n} \\
C_{1133}^{eff} &= C_{1122}^{eff}, C_{2222}^{eff} = C_{1111}^{eff}, C_{2233}^{eff} = C_{1122}^{eff}, C_{3333}^{eff} = C_{1111}^{eff} \\
C_{2323}^{eff} &= \frac{482.92}{30.43(1-n) + 15.87n} = C_{1313}^{eff} = C_{1212}^{eff}
\end{aligned} \quad (25)$$

The formula of volume average method for the five independent elements (C_{1111}^{eff} , C_{1122}^{eff} , C_{1133}^{eff} , C_{3333}^{eff} , C_{2323}^{eff}) in the Eq.(22) are presented in the following:

$$\begin{aligned}
C_{1111}^{eff} &= 37n + 24.47(1-n) = C_{2222}^{eff} = C_{3333}^{eff}, C_{1122}^{eff} = 6.52n + 8.60(1-n) \\
C_{1133}^{eff} &= C_{1122}^{eff}, C_{2233}^{eff} = C_{1122}^{eff}, C_{1212}^{eff} = 15.87(1-n) + 30.43n \\
C_{2323}^{eff} &= C_{1313}^{eff} = C_{1212}^{eff}
\end{aligned} \quad (26)$$

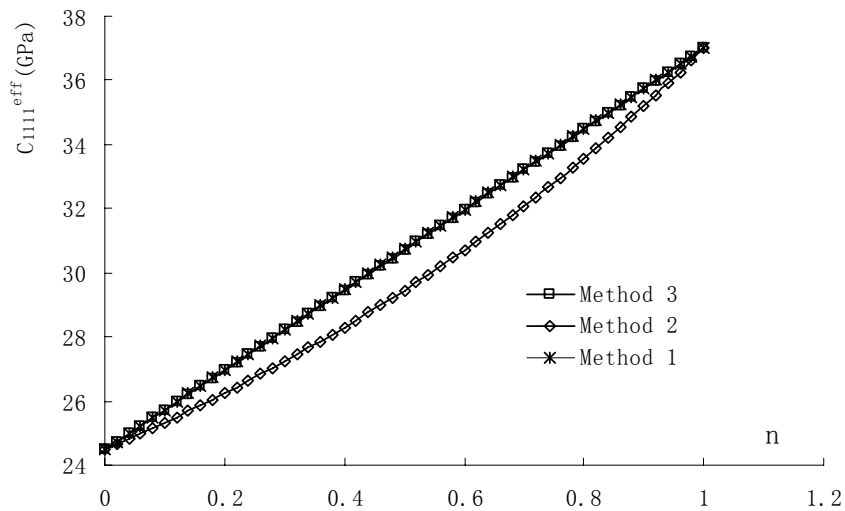


Fig.2 The comparative results of three sorts of methods for C_{1111}^{eff}

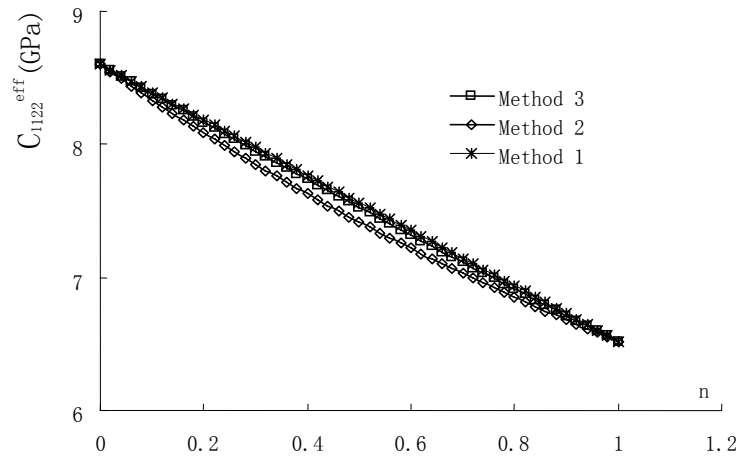


Fig.3 The comparative results of three sorts of methods for C_{1122}^{eff}

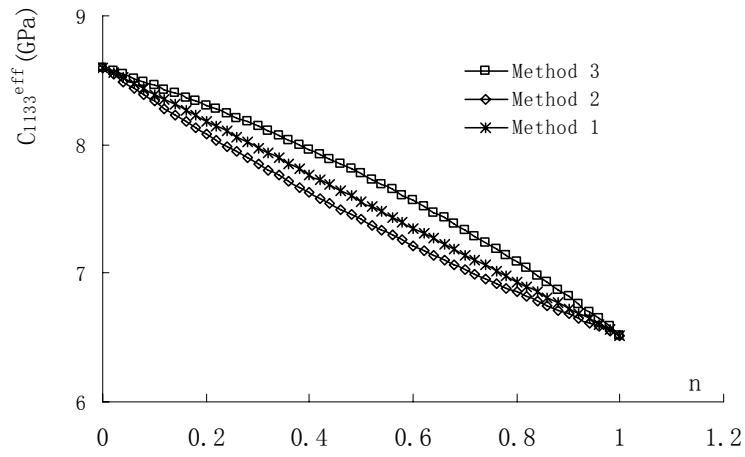


Fig.4 The comparative results of three sorts of methods for C_{1133}^{eff}

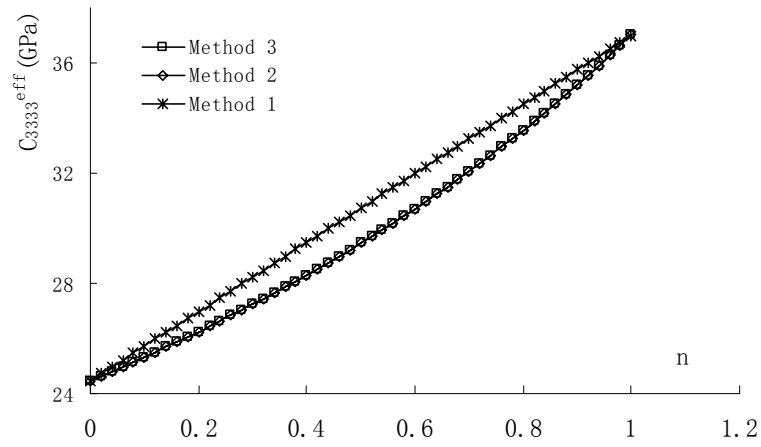


Fig.5 The comparative results of three sorts of methods for C_{3333}^{eff}

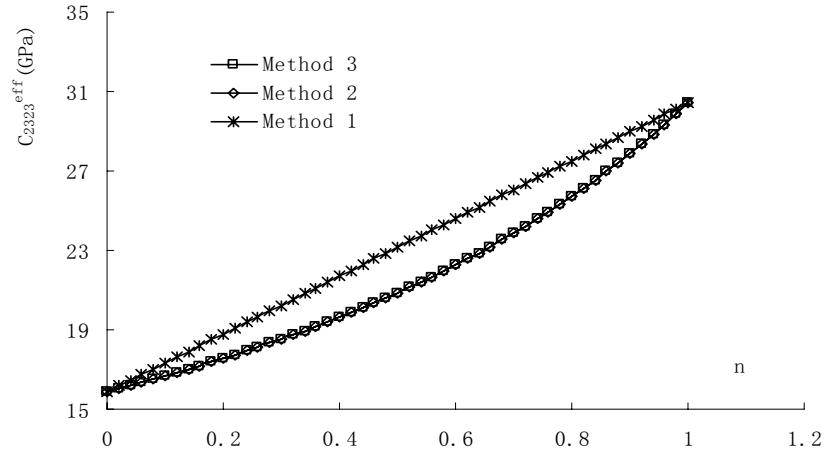


Fig.6 The comparative results of three sorts of methods for C_{2323}^{eff}

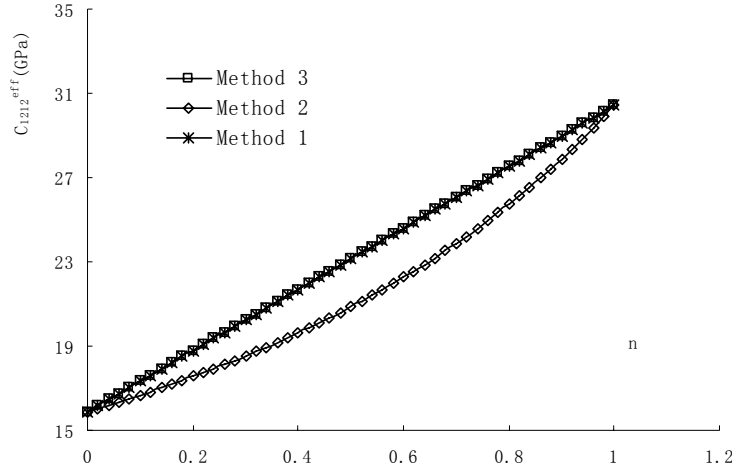


Fig.7 The comparative results of three sorts of methods for C_{1212}^{eff}

The calculating results are plotted on the Fig.2~7, the difference between method 3 and method 1 is not apparent, but it is very important between the method 1 (or method 3) and method 2 for the coefficients C_{1111}^{eff} and C_{1212}^{eff} . The result of method 2 is similar with the method 3, but the numerical result is very important between method 1 and method 2 (or method 3) for the coefficients C_{3333}^{eff} and C_{2323}^{eff} . The curves of method 2 and method 3 are convex and concave respectively, and lay over and below the curve of method 1 for the parameter C_{1133}^{eff} . The curves of method 2 and method 3 are convex, and lay below the curve of method 1 for the coefficient C_{1122}^{eff} , but the difference is important between method 2 (method 3) and method 1. The variable trends are shown on the Fig.2~7. The relationships of the coefficients C_{1111}^{eff} and C_{1212}^{eff} are in the good agreement with the series flow model, the theoretical results are proved with testing data in the special conditions. The relationships of the coefficients C_{3333}^{eff} and C_{2323}^{eff} are correspondent with the parallel flow model, the relating results are proved with testing data in the special conditions. But the relating test data are not reported for the coefficients C_{1122}^{eff} and C_{2233}^{eff} .

5 CONCLUSION

The analyses and researches are performed for the effective macroscopic stiffness matrix of transversal isotropic media with layered geological material; the comparative results are obtained for method 1, method 2 and method3. The conclusions are described in the following form:

- 1) If the integrals of microscopic strain and micro-stress over the researched area are equal to zero

respectively, and the stress is equal everywhere in RVE, the self-consistent method is obtained from the proposed theory in this paper. When the micro-stress is neglected, the simple volume average method (method 1) can be obtained to evaluate the macroscopic stiffness matrix.

2) The method proposed in this paper can describe the transversal isotropic media with layered geological material, the independent coefficients are five, and it is correspondent with the traditional theoretical deduction. In fact the independent parameters are three for the self-consistent method, and the independent parameter is one for the simple volume average method. However the five independent parameters are rational in the theory.

3) The difference between method 1 and method 3 is little for the coefficients C_{1111}^{eff} and C_{1212}^{eff} , the theoretical results are correspondent with parallel – flow model (Omr T. Frouki, 1991), but their numerical results are upper limited value. The theoretical result of C_{3333}^{eff} is correspondent with value of C_{2323}^{eff} , the theoretical results are in the good agreement with series – flow model [15], but their numerical results are lower limited value. The method proposed in this paper has been proved at least for the three independent coefficients (C_{1111}^{eff} , C_{3333}^{eff} , C_{2323}^{eff}) by tests, the parallel – flow model and series – flow model are special cases for the theory proposed in this paper.

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REFERENCE

- Cheng A.H.-D., *Material Coefficients of Anisotropic Poro-Elasticity, Int. J. Rock Mech. Min. Sci. Vol. 34, No. 2, P. 199-205, 1997.*
- Dragon A. Charlez P., Pham D., J.F. Shao, *Anisotropic Damage Model for Brittle Rock, -in Assessment and Prevention of Failure Phenomena in Rock Engineering, Pasamehmetoglu et al (ED.), Balkema, P. 71-78, 1993.*
- J.F. Shao, *Poro-elastic Behaviour of Brittle Rock Materials with Anisotropic Damage, Mechanics of Materials, Vol. 30, P. 41-53, 1998.*
- J.F. Shao, J.W. Rudnicki, *A Microcrack-based Continuous Damage Model for Brittle Geo-materials, Mechanics of Materials, Vol. 36, P. 52-63, 2000.*
- Kemeny J.M., *A Model for nonlinear Rock Deformation under Compression due to Subcritical Crack Growth, -Int. J. Rock Mech. Min. Sci., Vol. 28, P. 459-467, 1991.*
- Krajcinovic D., Fonseka G.U., *The Continuous Damage Theory of Brittle Materials, J. of Applied Mech., Vol. 48, P. 809-815, 1981.*
- Y.F. Lu, J.F. Shao, *Modelling of anisotropic damage in brittle rocks under compression dominated stresses. Int. J. Numer. Anal. Meth. Geomech. Vol. 26, P.230-247, 2002.*
- LU Yingfa, Zheng Junjie, Chai Huayou, *Characteristics of skempton Coefficient of Representative Rock and Soil, Chinese Journal of Rock Mechanics and Engineering, 2005, Vol. 24. No.11. PP. 1847-1851. (in Chinese).*
- Lu Yingfa, Tian Bin, Yu Tiantang, Shao Jianfu, *Testing Study of Mechanical Behaviors of Saturated Rock under Different Loading Path, Chinese Journal of Rock Mechanics and Engineering, 2005, Vol. 24, Supp.1. PP.5065-5071. (in Chinese).*

LU Ying fa, Zhou Shunpei, Tian Bin, Shao Jianfu, *Energy principal Of Rock and Water Interaction and Biot Coefficient* , *Chinese Journal of Rock and Soil Mechanics*, 2005 , Vol..26, Supp. 2. PP. 69-72. (in Chinese).

James G. Berryman, *Effective Medium Approximation for Elastic Constants of Porous Solids with Microscopic Heterogeneity*, *J. Appl. Phys.* Vol. 59(4), P. 1136-1140, 1986.

J.-L. Auriault and C. Boutin, *Deformable Media with Double Porosity. Quasi-static^{II}: Coupling Effects* , *Transp. Porous Media*, Vol. 10, P. 153-169, 1990.

Kachanov M., *Effective Elastic Properties of Cracked Solid: Critical Review of Some Basic Concepts*. *ASME, Appl. Mech. Rev.* Vol. 45(8), P. 304-335, 1992.

Kachanov M., *Elastic Solids with Many Cracks and Related Problems*, in *Advances in Applied Mechanics*, Vol. 30, Hutchinson and Wu (eds), NY 259-445 1993.

Omr T. Frouki, *Thermal Properties of Soils*, ELSEVIER SCIENCE PUBLISHERS B.V., Sr Burgerharststraat 25, P.O. Box 211, 1000 AE Amsterdam, the Netherlands, 1991.