

Interval Entropy Measurement Method in Risk Analysis and Its Application to Metro Construction

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ABSTRACT: Risk is related to uncertainty of potential events. Entropy is defined as the measure of uncertainty of a system. In underground engineering, one of definitions of risk is the probability of an event occurring. So, using entropy theory for risk measurement is a good way to evaluate the probability of a hazard in geotechnical risk system. In this paper based on the real practice of risk assessment and the difficulty in quantitative risk measurement, the interval entropy measurement method has been put forward. As an example, the risk of a shield tunnelling in construction stage is evaluated and determined using proposed method. In the end some conclusions had been drawn as well.

1 INTRODUCTION

Urban metro engineering is comprehensive system engineering with huge investment, long construction stage, many speciality combinations and involving many contractors. In construction stage, inadequacy of geotechnical information, geological and hydrogeological conditions locally different from those foreseen for the design, inappropriate choice of the construction method, non ideal performance or behaviour of the shield machine itself or deviation of the actual ground-machine system behaviour from the theoretical one, unfavourable ground environment, such as busy traffic, existing buildings/utilities and construction parameter and construction management and other uncertain factors, they result in many construction events or hazards (Grasso, et.al, 2007). Building owner and contractors bear much loss and how to evaluation risk and manage the risk in metro engineering to decrease the hazards loss is the main topic of urban tunnel works construction.

Since 1970's, underground engineering risk has been recognized and some research results have been obtained (Einstein, 1974). But, these results mainly focused on establishing conception of risk and some qualitative analysis. However the quantitative research work always focused on reliability calculation. In fact, reliability and risk in geotechnical engineering has inherent difference. Except these, other risk measurement method seldom involved.

In recent years, the importance of geotechnical construction risk has been realized increasingly. As Sir Michael Latham (1994) said "No construction project is risk free. Risk can be managed, minimized, shared, transferred or accepted. It cannot be ignored" (Hu, 2006). And many common risk analysis method including Delphi Method, Analytical Hierarchy Process, Influence Diagram Method, Monte-Carlo Simulation Method, CIM Model, Expected Value, Decision Tree Method, Fault Tree Method, Sensitivity Analysis Method etc and their combination have been developed(Chen, 2004). But, a suitable measurement method in underground engineering risk analysis has not been established. Thus, how to measure the risk using these common methods or

establish new quantitative measurement method is an important work in risk measurement now. In this paper, based on the idea of entropy and its generalization and extension, the interval entropy measurement method based on expert investigation was proposed.

2 CHARACTERISTICS OF URBAN SHIELD TUNNELLING RISK

Risk is related to uncertainty of potential events. Entropy is defined as the measure of uncertainty of a system. In underground engineering, one of definitions of risk is the probability of an event occurring, so using entropy theory for risk measurement is a good way to evaluate the probability of a hazard in geotechnical risk system.

In risk analysis practice of tunnelling construction of metro engineering, take shield tunnelling construction for example, because the likelihood of hazards occurrence is always difficult to determine owing to the lack of statistic data, the expert investigation method and Delphi method are widely used in practice. As for expert investigation, giving a single probability value of a certain risk factor is difficult for an expert. However, giving an interval value is less difficult than giving a single value to describe the probability occurrence of identified risk factor. Moreover, the interval value could reflect the dynamic risk management process that continues throughout the construction stage of the project system.

In this paper, based on the idea of entropy, interval entropy measurement method is put forward to evaluate the risk of events and a generalized definition of entropy is defined further, which is suitable for interval-valued entropy solution.

3 ENTROPY AND INTERVAL ENTROPY MEASUREMENT METHOD

3.1 Entropy and Risk

The idea of entropy is originated from thermodynamics, which is a physical quantity used to indicate that the process of thermal motion is irreversible. In 1929, L. Szilard, a Hungary scientist proposed the uncertainty relationship between entropy and information, which makes it possible to cite entropy in information science. In 1948, C. Shannon created the information theory in Bell laboratory (Shannon,1949). In information theory, the average information quantity of information source signal in communication process is called entropy, which made the information entropy use in practice. According to the C. Shannon theory, as for the discrete memoryless information source X , its information quantity i.e. entropy H is defined as a function of the n -dimensional probability vector $\vec{P} = (p_1, p_2, \dots, p_n)$, i.e.:

$$H(X) = -\sum_{i=1}^n p_i \log_2 p_i = H(\vec{P}) \quad (1)$$

$$\sum_{i=1}^n p_i = 1 \quad (2)$$

Entropy is the basic notion in the information theory field. Informally we can define entropy as the measure of uncertainty of a system that at a given moment can be in one of the states. Usually risk is related to uncertainty of future events. In other words, risk is related to lack of knowledge about future events. It would be natural to define risk as the amount of lacking information.

3.2 Interval-Valued Entropy

If values of all p_i are known, then entropy of a system can be calculated according to the formula (1). But in fact, the probability of the system in i^{th} state is always difficult to determine. Sometimes it is interval valued with $[p_i^{\min}, p_i^{\max}]$ rather than single valued. If probabilities are interval-valued, how can we calculate the entropy? Obviously, the entropy itself will be interval. According to formula (2), their probabilities should satisfy with equation (3)

$$\sum_{i=1}^n p_i^{\min} \leq 1 \leq \sum_{i=1}^n p_i^{\max} \quad (3)$$

We cannot use formula (1) directly to calculate entropy, as the function $h=-p \log p$ is not monotonic. Alexander Valishevsky proposed the calculation method of entropy for a system with interval-valued in 2003. The formula as follows (Aleksandrs, et.al, 2003):

$$H^{\max} = -\sum_{i=1}^n p_i^{\text{avg}} \log p_i^{\min} \quad (4)$$

$$H^{\min} = -\sum_{i=1}^n p_i^{\text{avg}} \log p_i^{\max} \quad (5)$$

Here, p_i^{avg} is defined as average probability shown in equation (6), which is obtained from maximum and minimum probability values.

$$p_i^{\text{avg}} = \frac{p_i^{\min} + p_i^{\max}}{2} \quad (6)$$

From equation (4) and (5), entropy for a system with interval-valued probability is interval entropy, which is equal to (7)

$$H = [H^{\min}, H^{\max}] \quad (7)$$

Thus, the interval-valued $H = [H^{\min}, H^{\max}]$ donates the probability of occurrence of system evaluation indexes. After the potential consequence of evaluation indexes and their weights are determined. The risk value could be calculated finally.

3.3 Interval Entropy Measurement Method in Risk Analysis

In city metro construction stage, many uncertainty factor results in much geotechnical risk. As we known that entropy is a measurement method for future information uncertainty and risk is related to uncertainty of future event. In other words, risk is related to lack of knowledge about future events. Naturally, entropy would be used to measure risk. As motioned above, interval entropy is a good way to measure geotechnical risk in metro construction stage. The idea of it follows:

(1) Firstly, according to the expert investigation method, every probabilities of risk indexes during metro construction stage are given, which are interval-valued with $[p_{ij}^{\min}, p_{ij}^{\max}]$.

(2) Supposing the importance degree of every expert is equal, using equation (4) and equation (5), calculate the interval entropy of every risk index $H = [H_i^{\min}, H_i^{\max}]$.

(3) Determine the entropy weight of every index. As for the multi-index making decision problem, the relative importance degree is needed to take into consideration. According to the concept of entropy, the information quantity and quality of every index provided for evaluation scheme, decides its relative weight in decision making. If the evaluation values of the j^{th} index are the same for every scheme, the entropy will reach maximum unit and entropy weight is zero. That entropy weight is equal to zero means the evaluation index doesn't provide any useful information for decision maker and decision maker would not consider it any more. However, if the evaluation values of the j^{th} index for every scheme have much difference, the value of entropy will be small and entropy weight will be large. In this case, evaluation index provides useful information for decision maker and this index should be given much emphasis on. Here, the entropy weight is not fixed for every evaluation index, and they would change with the change of evaluation scheme. Therefore, using entropy weight to describe the importance of every index is a reasonable rule

which embodies the relative importance of every index. The entropy weight should be determined according to equation (8):

$$\omega_i = \frac{1 - H_i}{m - \sum_{i=1}^m H_i} \quad (8)$$

(4) According to equation (9) and the entropy weight of every index, the risk of all risk factors in construction stage is:

$$R = \sum_{i=1}^m \omega_i H_i \quad (9)$$

where, ω_i is expert weight value of the i^{th} risk factor, which could be determined by expert investigation method according to equation (8). H_i is the entropy of the i^{th} risk factor, which could be given through the equation (7), and m is the number of risk factors.

The risk value obtained from equation (9) has the following properties:

- 1) $R \in [0,1]$, which could be proved by the feature of $\sum \omega_i = 1$ and $0 \leq H_i \leq 1$.
- 2) When R is little (or near zero), it means that the risk assessment system is almost definitive and there is no risk.
- 3) When R reaches up to maximum unit 1, it means that the risk assessment is carried out with minimum information. At this condition, the entropy of all H_i are equal to 1. This means that the risk factor values of all system are the same and any sequence shield construction will take risks. At this time, it is necessary to get much more the information, such as additional exploration survey, environmental investigation, training shield driver and inspection shield machine again etc..
- 4) The entropy can measure the quantity of useful information from available information.

4 CASE STUDY

In geotechnical risk analysis, a proposed way of assessing frequency is to have a risk assessment team, consisting of experienced tunnel engineers to formulate their own guidelines for frequency classes. These could be related to the number of events experienced by the participants, the number of events they have heard of, the number of experienced near-misses and the number of near-misses they have heard of; all in relation to the number of projects they have been involved in or are aware of. It would be of great benefit for a risk analysis (Aleksandrs, et.al, 2003). This analysis method is called expert investigation method. If the classifications of frequency of occurrence with interval values have been given, how we can determine the global risk level of the shield tunneling construction system is the focus of this paper.

In urban areas, tunnel works construction of is often associated with risk that can arise from different factors. The main categorises of risk includes geological and hydrogeological risk, design risks, construction risks and the third party risk. The following items are the main risk sources in urban shield tunnelling construction:

- 1) Geological cavern ahead of shield tunnelling machine
- 2) Encountered barrier or isolated granite during shield tunnelling
- 3) Shield tunnelling through river or lake
- 4) Shield equipment failure in tunnelling toward working-well
- 5) Excavation face failure
- 6) Leakage of water or slurry between segments connection
- 7) Axial deviation of shield tunnelling
- 8) Influence of shield tunnelling construction on running metro lines
- 9) Shield tunnelling through/across bridge piles, building piles or important buildings/utilities

10) Influence of shield tunnelling construction on important pipelines or municipal road.

In the following section, based on these risk sources mentioned above, combining with expert investigation method, the interval entropy risk measurement method would be introduced in details. The step follows:

1) Using expert investigation method, according to the frequency classification in *Guidelines of Risk Management for the Construction of Subway and Underground Works* published by China Civil Engineering Society (CCES, Tongji University, 2007), the probability of risk is divided into five classes shown in Table 1. At the same time separation into 5 classes or intervals is generally recommended as a practical way of classifying frequency by International Tunnel Association (ITA) (ITA, 2004).

Table 1 Classification of frequency of occurrence

Class/grade	1	2	3	4	5
Descriptive frequency class	Very unlikely	unlikely	occasional	likely	Very likely
Interval probability	$P < 0.01\%$	$0.01\% \leq P < 0.1\%$	$0.1\% \leq P < 1\%$	$1\% \leq P < 10\%$	$P \geq 10\%$

Note: P is the frequency of occurrence.

Assume that above mentioned ten risk factors are composed of a risk scheme of shield tunnelling construction system. In this system, seven experienced shield tunnelling engineering experts are composed of a risk assessment team. According to the rule of expert investigation method, we carried out risk assessment of this system. The evaluation matrix described by interval probability value of ten risk factors given by seven experts is obtained:

$$P = [p_{ij}^{\min} \sim p_{ij}^{\max}]_{m \times n}$$

$$= \begin{bmatrix} 0.01 \sim 0.1 & 0.0001 \sim 0.001 & 0.0001 \sim 0.001 & 0.001 \sim 0.01 & 0.001 \sim 0.01 & 0.001 \sim 0.01 & 0.01 \sim 0.1 \\ 0.01 \sim 0.1 & 0.0001 \sim 0.001 & 0.001 \sim 0.01 & 0.01 \sim 0.1 & 0.0001 \sim 0.001 & 0.001 \sim 0.01 & 0.001 \sim 0.01 \\ 0.001 \sim 0.01 & 0.0001 \sim 0.001 & 0.01 \sim 0.1 & 0.001 \sim 0.01 & 0.0 \sim 0.0001 & 0.001 \sim 0.01 & 0.01 \sim 0.1 \\ 0.001 \sim 0.01 & 0.0001 \sim 0.001 & 0.0001 \sim 0.001 & 0.001 \sim 0.01 & 0.001 \sim 0.01 & 0.0001 \sim 0.001 & 0.0 \sim 0.0001 \\ 0.001 \sim 0.01 & 0.0001 \sim 0.001 & 0.01 \sim 0.1 & 0.001 \sim 0.01 & 0.0001 \sim 0.001 & 0.001 \sim 0.01 & 0.0001 \sim 0.001 \\ 0.001 \sim 0.01 & 0.0001 \sim 0.001 & 0.01 \sim 0.1 & 0.001 \sim 0.01 & 0.0001 \sim 0.001 & 0.001 \sim 0.01 & 0.1 \sim 1.0 \\ 0.001 \sim 0.01 & 0.0001 \sim 0.001 & 0.0001 \sim 0.001 & 0.01 \sim 0.1 & 0.0 \sim 0.0001 & 0.001 \sim 0.01 & 0.0001 \sim 0.001 \\ 0.01 \sim 0.1 & 0.0001 \sim 0.001 & 0.01 \sim 0.1 & 0.001 \sim 0.01 & 0.0001 \sim 0.001 & 0.0001 \sim 0.001 & 0.001 \sim 0.01 \\ 0.01 \sim 0.1 & 0.0001 \sim 0.001 & 0.01 \sim 0.1 & 0.01 \sim 0.1 & 0.001 \sim 0.01 & 0.0001 \sim 0.001 & 0.01 \sim 0.1 \\ 0.001 \sim 0.01 & 0.0001 \sim 0.001 & 0.01 \sim 0.1 & 0.01 \sim 0.1 & 0.001 \sim 0.01 & 0.0001 \sim 0.001 & 0.01 \sim 0.1 \end{bmatrix}$$

Here, m is the number of risk factor, n is the number of expert.

2) According to equation (4) and equation (5), if the expert weight is assumed equally, the interval entropy value of ten evaluation index (i.e. ten risk sources) could be expressed with maximum entropy $H_i^{\max}$ and minimum entropy $H_i^{\min}$:

$$H = [H_i^{\min}, H_i^{\max}] = \begin{bmatrix} 0.0209 & 0.0391 \\ 0.0209 & 0.0391 \\ 0.0207 & 0.0388 \\ 0.0055 & 0.0080 \\ 0.0133 & 0.0237 \\ 0.0130 & 0.1020 \\ 0.0117 & 0.0214 \\ 0.0196 & 0.0371 \\ 0.0335 & 0.0658 \\ 0.0272 & 0.0525 \end{bmatrix}$$

3) Using formula (8), the entropy weight of ten indexes ω_i ($i=1, 2, 3 \dots 10$) are:

$$\omega_i(H^{\min}) = \begin{bmatrix} 0.100379215 \\ 0.100379215 \\ 0.100412047 \\ 0.103629626 \\ 0.101988004 \\ 0.093812727 \\ 0.102234248 \\ 0.100592626 \\ 0.097588457 \\ 0.098983836 \end{bmatrix}, \quad \omega_i(H^{\max}) = \begin{bmatrix} 0.099767892 \\ 0.099767892 \\ 0.099788999 \\ 0.101342209 \\ 0.100544497 \\ 0.100568516 \\ 0.10070171 \\ 0.099903998 \\ 0.098486894 \\ 0.099127393 \end{bmatrix}$$

4) Submit the ω_i and H_i ($i=1,2,3 \dots 10$) to equation (9), the risk interval value could be expressed by:

$$R = [R^{\min}, R^{\max}] = [0.0186, 0.0421]$$

In this evaluation project system, the risk of shield tunnelling construction is between 1.986% and 4.21%. It means the probability of occurrence of hazards in shield tunnelling construction lies in interval value [1.986%, 4.21%]. According to this risk ratio, the construction owner or contractors have a quantitative risk value for the whole construction. And the owner could use it to apply for insurance.

5 CONCLUSIONS

In this paper we presented the idea of using entropy as a measurement of risk, and based on the metro construction risk assessment practice and the difficulty, an interval-entropy measurement approach towards risk analysis is put forward. In this paper, we propose several heuristic approaches towards entropy generalization and extension, which are suitable to carry on quantitative risk assessment in metro construction system. Combined with the common assessment method, which is expert investigation method, it is believed that proposed quantitative measurement method is good way to measure the risk in metro construction.

It should be pointed that, there are many kinds of risk measurement methods. How to evaluate and analyze the risk much more effectively, feasibly is still a problem to be researched further.

REFERENCE

Aleksandrs Vališevskis, Arkady Borisov, (2003) Using Interval-Valued Entropy for Risk Assessment, *Proceedings of "Modelling and Simulation of Business Systems"*, Kaunas University of Technology Press–Technologija, Lithuania, 2003, pp.74-78.

Cheng Long, (2004) *Risk analysis and assessment during construction of soft soil shield tunnel in urban area*, [D], Tongji University, 2004.9, Shanghai, China.

China Civil Engineering Society (CCES), Tongji University (2007), *Guidelines of risk management for the construction of subway and underground works*.p51.

Einstein.H.H, Vick.S.G, (1974) *Geological model for tunnel cost model*. *Proceeding of Rapid Excavation and Tunnelling Conference*, 2nd, 1974, pp: 1701-1720.

ITA , (2004) *Guidelines for Tunneling Risk Management*.

P.Grasso, E.Chiriotti, S.Xu, et., (2007) *Use of risk management plan for urban mechanized tunnelling projects: From the establishment of the method to the successful practice*, *ITA-AITES Word Tunnel Congress 2007 and the 33rd ITA-AITES General Assembly*, 2007,5, Prague, Taylor & Francis Group, London, pp:1535-1540.

Qunfang Hu, (2006) *Risk Analysis and Its Application for Tunnel Works based on Research of Stratum and Soil Spatial Variability*, [D], Tongji University, 2006.9, Shanghai, China.

Shannon C., Weaver W. (1949), *The mathematical theory of communication*, Urbana: University of Illinois Press.

