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Evaluation of potential danger of slope failure using FEM-SS method

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ABSTRACT: This paper presents the FEM-SS method, in which FEM is used to evaluate potential danger of slope failure in risk management. Firstly, definition of Safety Factor of Potential Slip-surface (SFPS) was discussed. In order to clarify the problems associated with traditional Limit Equilibrium Method (LEM), the numbers of indeterminate problem and the assumptions in each LEM were discussed. Then, after an outline of the FEM-SS method is introduced, the method was verified using 'the standard slope', and slopes with different shape are analyzed using the LEMs and the FEM-SS method. It is concluded that the FEM-SS method can lead to same safety factor as those of Bishop method or the Modified Janbu method. Furthermore, it is suitable for use in risk management based on advantages that it can be used to evaluate the influences of slope deformation and complicated boundary conditions properly.

1 INTRODUCTION

In risk management, results of the slope stability analysis, namely evaluation of potential danger of slope failure, must be generality and validity. Limit equilibrium methods (LEM), such as the Swedish method, Bishop's method (1955) etc are widely used in design and analysis of slope stability. However, it is questionable whether LEMs may or may not be a suitable tool for evaluation of potential danger of slope failure. As is known, LEM equations involved hypotheses necessary due to the fact that the number of unknown variables is greater than that of known ones. Namely, it is an indeterminacy problem of LE equations, with the exception of stability analysis of the Infinite Slope Method or the Wedge shape slope method. The other problem is the progressive failure mechanism of a sliding mass, namely, the influence of slope deformation which is not taken into consideration in evaluation of slope safety. In addition, the influence of complex boundary conditions also can not be accurately evaluated in LEM analyses of a slope. The last problem, which is probably the most serious one, is that safety factor of a slope depends on the LE method adopted in analyses. Furthermore, a true safety factor of a slope is not known even when a LE method is used, which is regarded as the most reliable one, such as Janbu's method (1973) or Morgenstern & Price method (1965). Thus, uncertainties of LEM equations, which are mentioned above, directly resulted in reduction of the reliability of evaluation of risk when risk management for potential danger of slope failure is discussed.

As an alternative analysis method for evaluation of safety factor of a slope, a method using FEM was developed. Brown et al. (1966) reported the results of elastic FE analyses on a homogeneous slope. In the field of dam construction, Clough et al. (1967), Kulhawy et al. (1972) and Penman et al. (1973) presented results of deformation analyzed by FEM, which were compared with the data recorded from sites. However, deformation analysis based on elasticity theory could not sufficiently explain the failure mechanism of a dam/slope due to limitation of the methods. To evaluate safety factor of a potential slip surface is not easy by the analysis, and this approach is also not an adequate method to be adopted in the risk management.

For the other approach, FEM is also being used in order to evaluate stress distribution on slices slide surface on a sliding mass, which are produced in the same way as that of conventional slice methods. Such methods were proposed by Oden et al.(1971), Zou et al.(1995), Rahman(1996), Mochizuki et al.(1999) and Xiong et al.(2001). Advantages of the method based on FEM are that most problems of LEM mentioned above can be solved, and it may be considered that the method may satisfy conditions as a tool for risk management to evaluate potential danger of slope failure, though stress distribution on slices of a slope or deformation of it depends on stress-strain relationships used in FEM.

In this paper, firstly, in order to clarify the context of each LEM method the indeterminate number for each conventional LEM and the assumptions used in equations for a slope stability analysis are shown. Then, definition of safety factor of each LEM will be discussed. After that, outline of the FEM-SS method, which was developed in 1999 (Mochizuki, 1999), is introduced, and the method is verified using 'the standard slope'. Finally, safety factor of different shaped slopes, which are calculated by ordinary LEMs and the FEM-SS method, are compared, and the validity of the FEM-SS method as an adequate method to evaluate potential danger of slope failure will be discussed.

2 Definition of safety factor and indeterminate number of LEMs

Definition of safety factor on a potential slip surface (abbreviated as SFPS), is shown firstly in order to clarify a basic condition of the following discussion. Table 1 shows a list of definitions of SFPS. Symbols used in the table are explained below the table.

Table 1 definitions of Safety Factor on a Potential Slip-surface.

No.	A. M	Equation of F_s		Comments
1	LEM	$F_1=\Sigma S_f / W\cdot\sin\alpha$	(1)	Swedish method; Equilibrium of moment at a center of sliding is a base equation
2	LEM	$f_{2(i)}=\tau_f/\tau_{mob}$ $f_{2(i)}=f_{2(i+1)}=F_2$	(2)	Bishop's, Janbu's and M&S method
3	FEM	$f_3=R_2/R_1$ $f_{3(i)}\neq f_{3(i+1)}\neq F_3$	(3)	R_1, R_2: see Fig.1 Local safety factor is defined.
4	FEM	$f_{4(i)}=\tau_f/\tau_{mob}$ $f_{4(i)}\neq f_{4(i+1)}\neq F_4$	(4)	Local safety factor is defined.
5	FEM	$F_5=(\Sigma\tau_f\cdot\Delta l)/\Sigma\tau_{mob}\cdot\Delta l$ $(\tau_f/\Sigma\tau_{mob})_{(i)}\neq\tau_f/\Sigma\tau_{mob})_{(i+1)}$	(5)	Average safety factor of a potential slip surface is defined.

A.M.: Analysis method, LEM: Limit Equilibrium method, FEM: Finite Element method

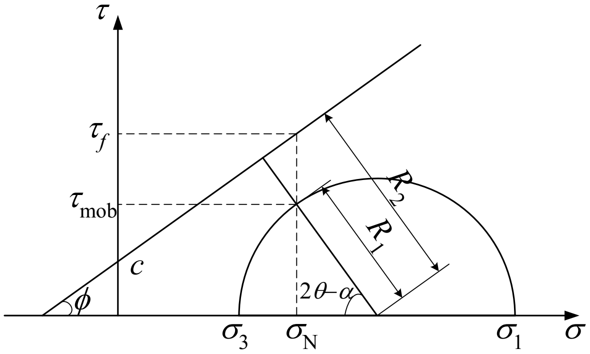

Fig. 1 Mohr's stress circle

Small form letter, *f*, means a local safety factor, and large form letter, *F*, is an average safety factor of a potential slip surface. S_f is given by $\tau_f\Delta l$, here τ_f is potential shear stress which is expected under *N* working vertically to a potential slip surface. An angle α is slope of a slice in a potential sliding mass. τ_{mob} is shear stress with the same magnitude, but opposite direction of sliding stress due to weight of a slice. Δl is length of a slice. R_i in equation (3) is lengths which are shown in Fig.1.

Equations (1) and (2), which are the definitions belonging to LEMs, give averages of SFPS. It should be noted that the widely used Swedish method adopt a balance of the moment at the centre of sliding rotation (Eq. (1)). All LEM except the Swedish method adopt definition of Eq. (2).

On the other hand, Eqs. (3) and (4), which are used in FEM, give local safety factors on a partial length of potential slip surface, which means that influence of deformation of a sliding mass is taken as evaluation of safety factor into consideration. This is one of the important features in evaluation of safety factor using FEM.

Eq. (5) gives an average SFPS similar to that of Eq. (2). However, the definition of SFPS by Eq.(5) is fundamentally different from that of Eq. (2), because Eq. (2) assumes a general failure of a potential sliding mass instead of a progressive failure in Eq. (5). In risk management, a method used to evaluate SFPS of a slope should be expressed with a definition of SFPS.

Next, Table 2 shows the numbers of known and unknown variables for each LEM.

Table 2 known-unknow conditions and degree of redundancy of the LE-methods

stablity computation	In-sl	Swedish	Bishop	GJ	M&P	MJ	ALE
shape of slip surface	P	C	C	NC	NC	NC	NC
number of slice	1	n	n	n	n	n	n
Known conds.							
1. Force acts on slice							
(1) W	1	n	n	n	n	n	n
2. Boundary conds.							
(1) $V_1, V_{n+1}=K_1$	-	2	2	2	2	2	2
(2) $H_1, H_{n+1}=K_2$	-	2	2	2	2	2	2
(3) $h_{t1}, h_{tn+}1=K_3$	-	-	-	2	2	2	2
unknown conds.							
3. Safety factor							
(1) F	1	1	n	n	n	n	n
4. Total strength							
(1) S	1	n	n	n	n	n	n
5. Force acting on slice							
(1) N	1	n	n	n	n	n	n
(2) T	1	n	n	n	n	n	n
(3) V	-	n-1	-	-	-	-	-
(4) H	-	n-1	-	-	-	-	-
(5) DV	1	-	n	n	n	n	n
(6) DH	1	-	n	n	n	n	n
(7) h_t	-	-	-	n-1	n-1	n-1	n-1
number of unknowns	6	5n-1	6n	7n-1	7n-1	7n-1	7n-1
cond. of redundancy							
6. Equilibrium of slice							
(1) horizontal direction or slip surface direction	1	n	n	n	n	n	n
(2) vertical direction or normal to the slip surface direction	1	n	n	n	n	n	n
(3) moment	-	-	-	n	n	n	n
7. Equilibrium of inner slice force							
(1)SDH=0	1	-	-	1	-	1	1
(2) SDV=0	1	-	-	-	-	-	-
8. Total equilibrium							
(1) H=0	-	-	-	-	-	-	-
(2) V=0							
(3) M=0	-	(1)	1	-	-	-	-
9. Strength formula							
(1) Coulomb formula	1	n	n	n	n	n	n
10. Formula of safety factor							
(1) $F=M_R/M_D$	-	1	-	-	-	-	-
(2)F=S/T	1	-	n	n	n	n	n
(3)$F_1=F_2=...=F_n$	-	-	n-1	n-1	n-1	n-1	n-1
total of formulas	6	3n+1	5n	6n	6n-1	6n	6n
degree of redundacy	0	2n-2	n	n-1	n	n-1	n-1

Modified Janbu method in the table was presented by Mochizuki (1986). Number of indeterminate equations, which is obtained by subtracting the total of known formulas from that of unknown formulas, is shown in the bottom row of the table. The number of indeterminate equations is not zero except in the case of Infinite Slope analysis, which means that other LE methods are all not able to be solved without assumptions whose number is the same as that of indeterminate variables. Table 3 shows assumptions which are used in each LEM. The last column of the table shows number of assumed equations. It can be seen that the number of Table 3 coincides with that of the bottom row in Table 2, which results in each LEM system becoming stable.

Table 3 Assumptions used in the LE-methods

method	shape of slip	assuming	number of assumption
Swedish	circular	V=0 H=0	n-1 n-1 total 2n-2
Bishop	circular	V=0	N
Original Janbu	circular & noncircular	assumed relevant ht	n-1
M&P	circular & noncircular	$V=f(x)H$: n-1 of $f(x)$	n-1 1 total n
Modified Janbu	circular & noncircular	$h_t=h_s/3$	n-1
Advanced Limit Equilibrium	circular & noncircular	$V/H=-\tan\beta_i$	n-1

Needless to say quality or accuracy of each LE method directly depend on validity of assumed equations listed in Table 3.

It should be understood that the Original Janbu method and M & P method require supplement any assumptions in order to solve equations of the methods besides assumptions shown in Table 3. This would be problematic if the methods were adopted in risk analysis. Mochizuki et al. (1999) proposed an additional assumption in Janbu method, referred as Modified Janbu method, which is the only method able to be applied to analyse safety factor of non-circular slip surface practically. Derivation of Modified Janbu method is shown Appendix 1.

However, one problem is that safety factors obtained using any type of LEM in the table could not prove that the safety factor obtained using a LEM would be lower/higher than an actual value if it were able to be obtained. This means that a LE method, even Modified Janbu method or M & P method, can not be adopted as a method which presents a reliable index for judging the risk of potential slope failure.

3 OUTLINE OF THE FEM-SS METHOD

Following the FEM scheme shown in Fig.2 vertical stress, σ_N, distributed on a partial length of a potential slip surface is obtained, and maximum shear resistance stress, τ_f, is calculated using Eq.(6). Then, an average safety factor of a potential slip surface is found using Eq. (5) in Table 1. Mobilized share stress, τ_{mob}, is the same magnitude to the opposite direction of shear stress vector working on the partial length of a potential slip surface. Here, c and ϕ are strength parameters, and Δl is partial length of the potential slip surface concerned. Stress components, σ_N and τ_f, are obtained directly using a configuration matrix at point A, the middle of partial slip surface, of Eq. (7), and then Eq. (8) and (9) for increment stresses and strains.

$$\tau_f = (c + \tan\phi)\Delta l \qquad \textbf{(6)}$$

Δl: a partial length of a potential slip surface

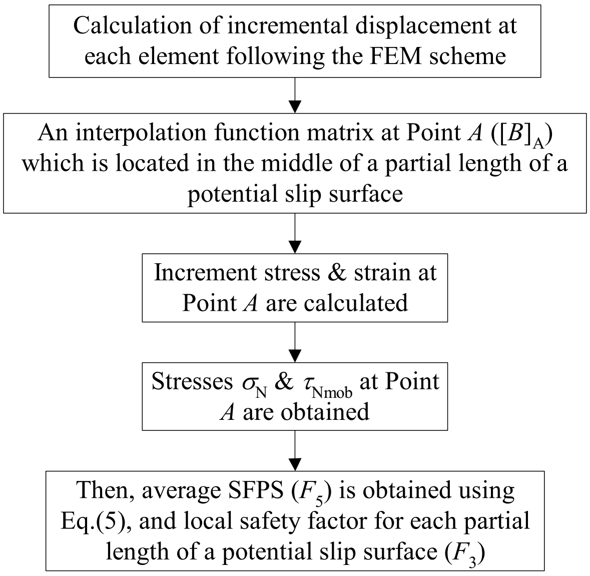

Fig.2 FEM scheme of FEM-SS

$$[B]_A = \begin{bmatrix} \frac{\partial N_i}{\partial X_A} & 0 \\ 0 & \frac{\partial N_i}{\partial Y_A} \\ \frac{\partial N_i}{\partial Y_A} & \frac{\partial N_i}{\partial X_A} \end{bmatrix} \tag{7}$$

(Here, i=1~4, and N_i is an interpolate function)

$$\{\Delta\varepsilon\}_A = [B]_A\{\Delta u\} \tag{8}$$

$$\{\Delta\sigma\}_A = [D]\{\Delta\varepsilon\}_A \tag{9}$$

Next, a couple of principal stresses and an angle of a potential slip surface at point A to the horizontal, α, are obtained, and that to σ_1, θ, are translated into stresses, σ_N and τ_f, using Eq. (10) and (11). As safety factor of a local potential slip surface is defined by Eq. (3) in Table 1.

$$(\sigma_N)_A = \frac{(\sigma_1 + \sigma_3)}{2} - \frac{(\sigma_1 - \sigma_3)}{2} \cdot \cos 2(\theta - \alpha) \tag{10}$$

$$(\tau_{mob})_A = \frac{(\sigma_1 + \sigma_3)}{2} \cdot \sin 2(\theta - \alpha) \tag{11}$$

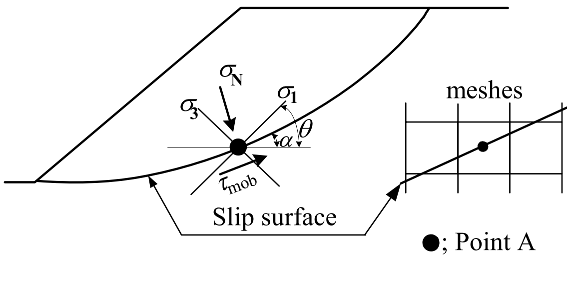

Fig.3 Principle stresses on slide surface

4 Verification of the FEM-SS method using the Standard slope

4.1 Comparison of safety factors

It was found that the FEM-SS method is advantageous over LE methods, as no assumption is introduced to solve it. However, it is not so simple if an average safety factor obtained by the FEM-SSM becomes significantly different to that by ordinary LE methods. In order to determine which analysis method should be adopted, verification of the FEM-SS was carried out using the Standard slope with a slope of 1:2. The slope is designed to have unity of safety factor of F_2 in Table 1 for the critical slip surface when it is analyzed using Bishop method with material parameters, c=3.0kPa, ϕ=20°and γ_t=20kN/m^3. In the FEM-SS method, homogeneous elastic material with 19.6MPa of Young's modulus, E, and 0.25 of Poisson's ratio, ν, is assumed. This analysis is named as Case 1.

Fig.3 shows the standard slope with locations of critical potential slip surfaces, and Table 4 shows their safety factors of the critical slip surface. In the table, F_{ZU} is a safety factor obtained by Zou et al. (1995) calculated by Eq. (5) in Table 1, in which elastic material was assumed. F_{ZK} is the safety factor calculated by Zienkiewicz et al. (1968), who decreased the strength gradually until when the solution can not converge, as it was defined as failure occurred.

The safety factor of the critical slip surface calculated by the FEM-SS method, F_{XM}, was 0.99 from about 1,600 trial circles, showing almost the same location as those of others in the slope. These show that the proposed FEM-SS method gives satisfactory results. A critical safety factor value of approximately 3% smaller than that of the Swedish method (last column of table 4) was obtained. Even when the method adopted rough assumptions as shown in Table 2, error of the SFPS is not bigger than that which was predicted.

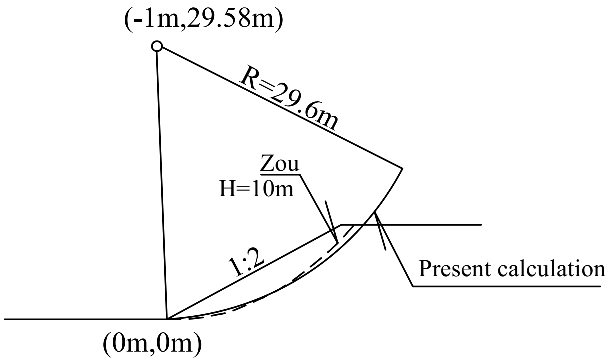

Fig.4 Critical potential slip surface and safety factors; Case 1

Table 4 Safety Factors of deferent methods

F_{ZU}	F_{ZK}	F_{XM}	F_1
1.01	1.0	0.99	0.97

F_{ZU}: by Zou etal., F_{ZK}: by Zienkiewicz
F_{XM}: by Xiong etal., F_1: by Swedish method

4.2 Influence of mesh size in the FEM-SS method

Table 5 shows safety factors of the standard slope with different mesh size. Number of meshes is shown upper row of the table. As it is shown, even when the meshes refined by 4 times, the safety factor of the potential slip surface changes only 1/500. Therefore, it may be said that influence of mesh size on safety factor is negligible in the FEM-SS method, in which an iso-parametric mesh with 4 nodes is used. This finding proves that the FEM-SS method can be used as an ordinary tool of slope stability analysis.

Table 5 Safety factor relate to mesh

Number of element	241	628	944
Safety factor	1.012	1.013	1.014

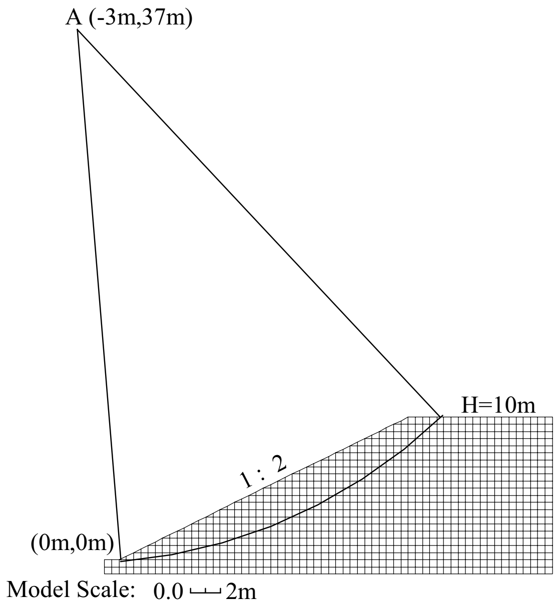

Fig.5 Critical potential slip surface and safety factors; Case 1

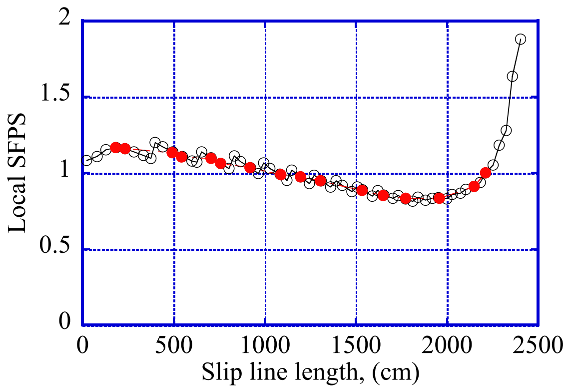

Fig.6 Critical potential slip surface and safety factors; Case 1

5 Stability analysis of slopes with different shapes

In order to verify properties of the FEM-SS method in calculation of SFPS, safety factors of slip surfaces of slopes in Case 2 to 4 (shown in Fig.5) were obtained using the FEM-SS method and the ot her traditional methods. Table 5 shows material properties of slopes.

Table 6 shows safety factor of each slip surface presented in Fig.5. Traditional methods used in the comparison were the Swedish method, Bishop method and the Modified Janbu method (see Table 2). In the FEM-SS method material of slopes were assumed to be elastic, in which Young's modulus is 49MPa and Poison's ratios are 0.25 or 0.4. As shown in the second column for results of ν=0.25 and the third column for those of ν=0.4 of Table 6. Influence on value of safety factor is as small as only ±1% of a ratio between safety factors for ν=04 against that for ν=0.25. This result shows an advantage of the FEM-SS method because Poisson's ratio is difficult to measure accurately by a laboratory test or field test.

When a SFPS of the FEM-SS method with a Poison's ratio of 0.25 is considered as an index safety factor to compare them, it is found that only +1% to +5% differences of safety factors for Bishop method and that of -1% to +3% for Modified Janbu method, though safety factors by the FEM-SS are +5 to +10% higher than those by the Swedish method. It should be noted that the rate of difference in a safety factor by the FEM-SS method depends on shape of a slope and boundary conditions. It should be noticed that the traditional LE methods present acceptable safety factors in practice, though each LEM introduced assumptions in equations in order to make the analysis systems to be statically. This result proves that traditional LEMs have a sufficient level of reliability in evaluating SFPSs, and that the FEM-SS method is quite useful and accurate enough in order to evaluate SFPS.

It is considered to be an advantage that the FEM-SS method presents almost the same value of SFPS as those of traditional LE methods, if those were to be replaced by the FEM-SS method as a standard method in the risk analysis system. It should be recognised that the FEM-SS method can be used to evaluate SFPS with complicated boundary conditions.

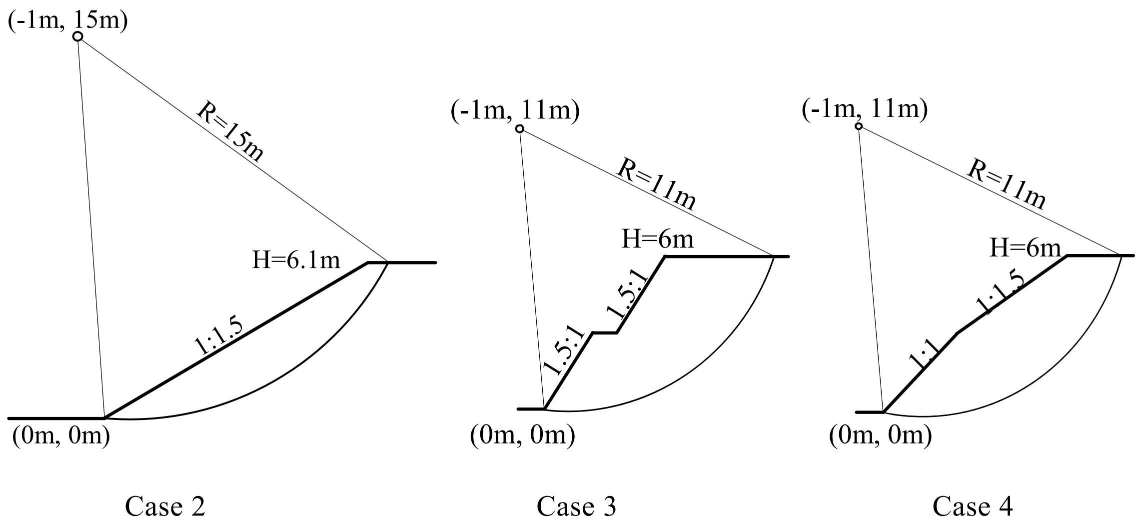

Fig.7 Statement of slope and shape of slip surface Case 2~4

6 CONCLUSIONS

The following was concluded from this study.

(1) Definition of safety factor for ordinary LE methods and for methods in which FEM is involved. In LE methods the general failure mechanism is assumed implicitly, whereas the progressive failure mechanism is taken into consideration in methods using FEM.
(2) LE methods, with the exception of Finite slope analysis and the Wedge method, are composed of indeterminable systems of stability analysis, which means that the LE methods necessarily involve the same number of assumptions as the indeterminable number of equations of stability analysis. Furthermore, Original Janbu method and M & P method require supplementary assumptions. From this discussion, it can be concluded that LE methods are not suitable for use in risk management.
(3) The FEM-SS method is introduced, and it is found that the method gives almost the same safety factor and location of critical slip surface as that by Bishop method when the 'standard slope' is analyzed. This result shows that it would be advantageous to use the FEM-SS method rather than other LE methods for risk management of potential slope failure.

(4) It was found that Poison's ration in elastic analysis of the FEM-SS method shows little influence on average safety factors of several slopes. This result shows another advantage of the FEM-SS method because Poisson's ratio is difficult to measure accurately by a laboratory test or field test.
(5) Four model slopes with different shape were analyzed by the traditional LE methods and the FEM-SS method. It was found that the FEM-SS method shows small difference of safety factors less than 10% as those obtained by other LE methods, in which the gap of safety factor to that by the LE methods depends on shape of a slope. It is concluded that the FEM-SS method is quite useful and sufficiently accurate in order to evaluate SFPS.

Appendix: Introduction of Modified Janbu's method*1

Fig.A-1 shows inter-slice forces acting on a slice in a soil mass on a potential slip surface. Neutral porewater pressures due to seepage flow, water table, or consolidation etc., are regarded as external pressures acting on the surface of each slice.

The force, *N*, acting normal to a potential slip surface is assumed to be acting on point *m*, where the vertical line from the centroid of a slice crosses the potential slip surface. As for seismic force, a lateral seismic coefficient, K_h, is considered in the quasi-static coefficient method.

The Modified Janbu method (referred to the MJ-method in this paper) used the formula for safety factor of a slope by equilibrium of inter-slice forces for both horizontal and vertical directions in addition to momentum equilibrium of forces at point *m*. Height of the thrust line (ht in Fig.1) is assumed to be 1/3 of a slice height in the MJ-method (Mochizuki et al., 1986). The derivation process of the MJ-method is basically the same as that of the Generalized Janbu method presented by Janbu (1973). Referring symbols are shown in Fig.A-1. Safety factor of a slice is defined by Eq.(A-1).

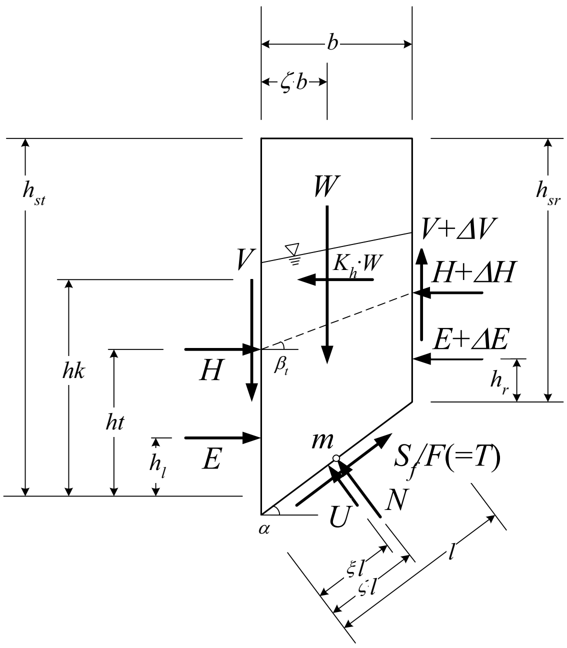

Fig. A-1

$$F_J = S_f / T \tag{A-1}$$

Here, S_f represents potential resistance force and *T* represents mobilized shear force on the slip surface which takes the same magnitude with an inverse sign of the sliding force. By considering equilibrium of forces in a direction parallel and normal to the potential slip surface, the following Eq.(A-2) is obtained:

$$\Delta H = \frac{cb + (W - \Delta V - U\cos\alpha)\tan\phi}{F_J(\cos^2\alpha + \sin\alpha\cos\alpha\tan\phi / F_J)} - \Delta E - K_h W - (W - \Delta V)\tan\phi \quad \text{(A-2)}$$

Inter-slice forces *ΔH* and *ΔV* in the above equation are shown in Fig.A-1. Taking *ΣH*=0 into consideration for all slices, Eq. (3) is given as follows:

$$F_J = A / \sum[(\Delta E + K_h W)\cos\alpha + (W - \Delta V)\sin\alpha] \quad \text{(A-3.1)}$$

$$A = \sum \frac{[cb + (W - \Delta V - U\cos\alpha)\tan\phi]}{[\cos\alpha + \sin\alpha\tan\phi / F_J]} \quad \text{(A-3.2)}$$

Moment equilibrium of forces around point *m* will give Eq. (A-4) with regard to *V* when Eq. (A-5) is assumed and *D* in Eq. (A-4) is neglected following Janbu guide in his original method.

$$V = -H\tan\beta_t - \frac{\Delta H h_t}{b} + E\left[\frac{(h_l - h_r)}{b} - \tan\alpha\right] - \Delta E\left[\frac{h_r}{b} + (1-\zeta)\tan\alpha\right] + \frac{U(\zeta - \xi)}{\cos\alpha} - K_h W\left[\frac{h_k}{b} - \zeta\tan\alpha\right] + D \quad \text{(A-4)}$$

Here, $D = -\Delta(1-\zeta) + \Delta H(\zeta\tan\alpha - \tan\beta_t) = 0$

Equation (A-5) is presented by Mochizuki (1986). Boundary conditions at both ends of a potential slip surface are shown in Eq. (A-6):

$$h_t = h_{st} / 3 \quad \text{(A-5)}$$

$$\begin{aligned} V_1 &= 0 & V_{n+1} &= 0 \\ H_1 &= 0 & H_{n+1} &= 0 \\ h_1 &= 0 & h_{n+1} &= 0 \\ E_1 &= 0 & E_{n+1} &= 0 \end{aligned} \quad \text{(A-6)}$$

The suffix of symbols in Eq. (A-6) denotes slice number. Then, we can obtain a safety factor of a potential slip surface when FJ on the left-hand side of Eq. (A3-1) coincides with the assumed FJ on the right-hand side of Eq. (A3-2) under the condition given in Eq. (A-4).

*1:This method was referred as 'Simplified Janbu method' in a previous paper.

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