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Effects of soil hydraulic hysteresis on slope reliability

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ABSTRACT: The hysteretic nature of soil hydraulic functions has been known for a long time. It can affect the soil atmosphere fluxes and consequently influence the hydraulic process in soil slopes and slope stability. Usually the soil hydraulic functions are assumed to be nonhysteretic in routine engineering practice and soil hydraulic functions measured in drying conditions are used. In this paper, the effects of hydraulic hysteresis on slope reliability during cycles of evaporation and precipitation are studied. Three cases of reliability analyses with different assumptions about the hysteresis of soil hydraulic properties are conducted for an unsaturated soil slope with completely decomposed volcanic soils. It is found that the variation of the mean and the COV of the safety factor with time are most significant in Case 1 where the drying hydraulic functions are used in both drying and wetting cycles. Without considering soil hydraulic hysteresis, the reliability of the slope is underestimated.

1 INTRODUCTION

The water content of a soil decreases with soil suction during drying process, while the water content increases when the soil suction decreases following a wetting path. The drying and wetting curves in general are not identical. This phenomenon is called hysteresis. The hysteretic nature of soil-water characteristic curve (SWCC) has been known for a long time. Hysteresis in the SWCC may be caused by non-uniformity of the pore cross-sections ("ink-bottle" effect), different contact angles during drying and wetting, entrapped air, swelling or shrinking and aging effect (Klausner, 1991). Hysteresis also has been found in coefficient of permeability functions of unsaturated soils (Fredlund and Rahardjo 1993). The hysteresis of soil hydraulic properties can affect the soil atmosphere fluxes and consequently influence the hydraulic process in soil slopes. Therefore, there is a need to study the effect of hydraulic hysteresis on slope reliability during cycles of evaporation and precipitation.

The objective of this paper is to study the effects of soil hydraulic hysteresis on reliability of soil slopes. Reliability analyses are conducted for an unsaturated soil slope with completely decomposed volcanic soils. A finite element unsaturated soil seepage analysis program together with a finite element based slope stability program are used as the deterministic model for reliability analysis. The basic statistics of the hydraulic property parameters of the completely decomposed volcanic soils in Hong Kong are used in the reliability analysis. Three cases of reliability analyses with various assumptions about hysteresis of soil hydraulic properties are conducted and the results are presented to demonstrate the effect of soil hydraulic hysteresis on slope reliability.

2 THEORY OF WATER FLOW IN UNSATURATED SOILS

2.1 Governing equations

Two-dimensional water flow through saturated and unsaturated soils is governed by the following equation:

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y \frac{\partial h}{\partial y}\right)=m_2^w \rho_w g \frac{\partial h}{\partial t} \tag{1}$$

where k_x = coefficient of permeability in the x-direction; k_y = coefficient of permeability in the y-direction; m_2^w = water storage coefficient which is the slope of the soil-water characteristic curve (SWCC, the curve of volumetric water content θ_w versus soil suction ψ); h = hydraulic head; ρ_w = density of water; g = gravitational acceleration and t = time.

2.2 Hydraulic functions and their hysteresis

The above governing equation indicates the rate and amount of water that can infiltrate into an unsaturated soil depends on the coefficient of permeability and the water storage coefficient of the soil, both of which are functions of soil suction. The relationship between the coefficient of permeability of an unsaturated soil and soil suction is called the permeability function. The coefficient of permeability of an unsaturated soil decreases with an increase in soil suction. The water storage coefficient, m_2^w, represents the water storage capacity of an unsaturated soil at any soil suction level. Both the permeability function and the water storage coefficient function are related with the soil-water characteristic curve. A number of equations have been suggested for SWCC (Leong and Rahardjo, 1997). One of the equations is the Fredlund and Xing (1994) equation:

$$\theta_w(\psi) = C(\psi)\frac{\theta_s}{\left\{\ln\left[e+\left(\frac{\psi}{a}\right)^n\right]\right\}^m} \quad (2)$$

where θ_w = volumetric water content; θ_s = saturated volumetric water content which is equal to porosity; a = matric suction value at the inflection point and is closely related to the air-entry value of the soil; n = the slope at the inflection point; and m = a fitting parameter related to residual water content, e = the base of natural logarithm. $C(\psi)$ is a correction function which makes the SWCC goes to zero when the soil suction is 10^6 kPa.

$$C(\psi) = 1 - \frac{\ln(1+\psi/\psi_r)}{\ln(1+10^6/\psi_r)} \quad (3)$$

where ψ_r is the suction corresponding to the residual water content.

Although the hysteretic nature of soil hydraulic functions has been known for a long time, the soil hydraulic functions are assumed to be nonhysteretic in routine engineering practice because measurement of complete set of hysteretic hydraulic properties is extremely time consuming and costly. Pham et al. (2005) compared several hysteresis models for SWCC and developed a simple scale method for estimating hysteretic SWCCs. In this following section, the findings from the study by Pham et al. (2005) are used to obtain a wetting SWCC from a drying curve.

3 RELIABILITY ANALYSIS OF A SOIL SLOPE

3.1 Model of an unsaturated soil slope

A 20 m high slope inclined at 30 degrees shown in Fig.1 is used in the reliability analyses. The initial pore-water pressure distribution in the slope is hydrostatic with respect to the assumed ground water table. A constant head is applied along each of the left and right boundaries beneath the ground water table. A zero flux boundary was applied along the left and right boundaries above the ground water table. The base of the finite element mesh was assumed to be impermeable.

Precipitation and evaporation is modelled by a moisture flux boundary applied along the slope surface. The precipitation is modelled as an influx boundary condition while the evaporation is modelled as a flux with negative value. Transient seepage analyses are conducted using the finite element seepage analysis software Seep/W. The boundary condition applied on the slope surface is assumed to be three cycles of precipitation and evaporation with the precipitation rate of 2.0×10^{-5}m/s and the evaporation rate of 1.0×10^{-6}m/s. In each cycle of precipitation and evaporation, the duration of precipitation is 12 hours and the duration of evaporation is 8 hours.

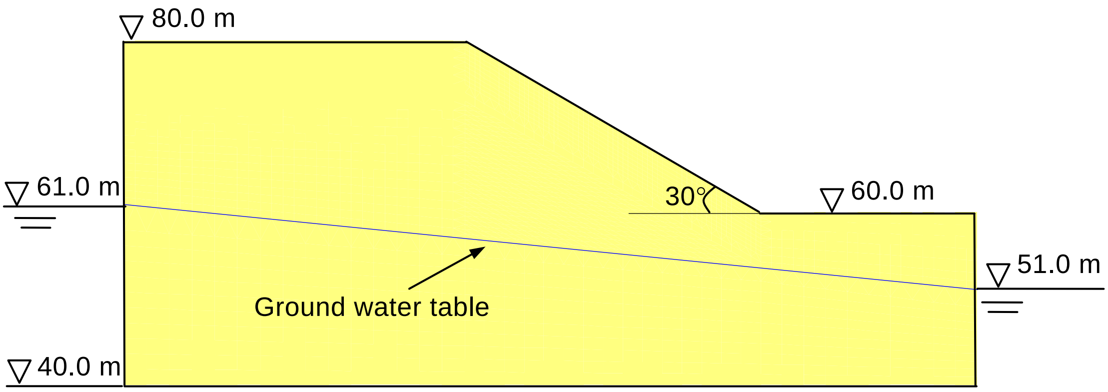

Fig.1 The profile of a soil slope for numerical modeling

3.2 Uncertainties of soil hydraulic properties

It is assumed that the soil slope in this study is a completely decomposed volcanic slope. Based on the laboratory studies on the soil-water characteristic curves of the decomposed volcanic soils conducted by Gan and Fredlund (1997) and Pang (1999), a statistical analysis for the parameters of the Fredlund and Xing (1994) SWCC model can be performed. Table 1 presents the basic statistics for the four parameters in the Fredlund and Xing SWCC model (Eq. 2). It can be seen the uncertainties of the parameters *a*, *n* and *m* are greater than the uncertainty of the saturated volumetric water content. Based on the previous study (Zhang et al. 2003), ln(a), ln(n), ln(m) and θ_s are believed to follow normal distributions. ln(k_{sat}) also follows a normal distribution. Table 2 summarized the mean, standard deviation and the distributions for the five hydraulic parameters. Table 3 presents the correlation coefficients among the parameters.

Table 1 Basic statistics of four parameters in the Fredlund and Xing equation for CDV soil

	θ_s	a	n	m
	0.381	38.502	2.147	0.628
	0.058	76.609	3.031	0.592
COV	15%	199%	141%	94%
Max	0.446	300.001	13.296	2.116
Min	0.275	0.983	0.487	0.097

Note: ψ_r = 3000 kPa is used when obtaining the best-fit parameters.

Table 2 Mean, standard deviation and distribution of random variables

			Distribution
θ_s	0.381	0.058	Normal
ln a	2.363	1.593	Normal
ln n	0.281	0.888	Normal
ln m	-0.842	0.896	Normal
ln(k_{sat})	-11.6	0.791	Normal

Table 3 Correlation matrix of the random variables

	θ_s	ln *a*	ln *n*	ln *m*	$\ln(k_{sat})$
θ_s	1				
ln *a*	-0.19	1			
ln *n*	0.10	-0.57	1		
ln *m*	-0.31	0.83	-0.65	1	
$\ln(k_{sat})$	0.74	-0.19	0.30	0.28	1

3.3 Numerical analyses and reliability study

In this study, three cases of reliability analyses are conducted with different assumptions about the hysteresis of SWCCs. In Case 1, the hysteresis of SWCC is not considered. The drying curve of SWCC is used in the seepage analysis as in common engineering practice. In Case 2, the hysteresis of SWCCs is considered by using the drying and wetting SWCCs in the evaporation and precipitation conditions, respectively. According to Pham et al. (2005), the ratio between the slope of the boundary drying curve and the slope of boundary wetting curve can be assumed to be 1.0 and the distance between the boundary drying and wetting curves is 0.2 for sand in the semilogarithmic coordinate system. In this study, the ratio of the parameter *a* in the Fredlund & Xing (1994) SWCC model between the wetting and drying SWCCs is assumed to be 0.2 while the parameters *m* and *n* of the wetting curves are assumed to be the same as those of the drying curves. It is also assumed the saturated permeability coefficient of a wetting permeability function is the same as that of the drying permeability function. In Case 3, the hysteresis of SWCCs is simplified by using the average curve of the drying and wetting curves.

In each case of reliability analysis, the Latin Hypercube Sampling technique is adopted to generate random samples of the five random variables according to Table 2 and 3. The coefficients of permeability functions can be estimated from the SWCC and the generated sample of k_{sat} using the Fredlund et al. (1994) prediction method. It should be noted that the experimental SWCCs of the completely decomposed volcanic soils are all drying curves. Hence, the generated SWCCs are all drying SWCCs and are used in Case 1 directly. While in Case 2 and 3, the wetting SWCCs and the average SWCCs are obtained from the drying curves accordingly. In the stability analysis of the unsaturated soil slope, it is assumed the unit weight of the CDV soil is 18 kN/m^3. The friction angle ϕ and the cohesion of the soil is 35 degree and 2 kPa. The friction angle related with the matric suction, ϕ^b, is assumed to be 20 degree.

3.4 Results and discussion

Fig.2 presents the variation of mean of safety factor during three cycles of precipitation and evaporation. It shows that for all the three cases the mean of safety factor decreases during the precipitation and increases during the evaporation. After the three cycles of precipitation and evaporation, the mean of safety factor is smaller than the initial mean of safety factor. It is found that the variation of the mean safety factor with time is most significant in Case 1 where the drying hydraulic function is used. The mean safety factor of Case 2 is largest among all the three cases. Fig.2 illustrates that the use of drying hydraulic functions yields a smaller mean value of the safety factor of the slope.

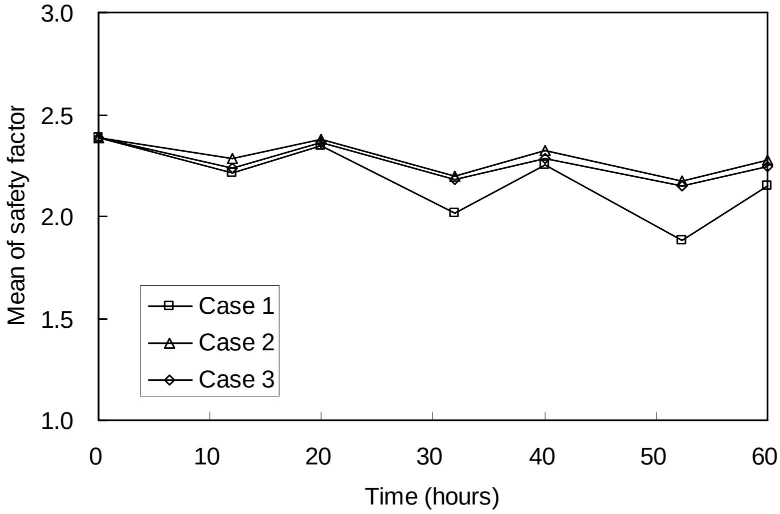

Fig.2 Variation of mean of safety factor with time

Fig.3 shows the variation of COV of safety factor with time. For all three cases, the COV of the safety factor is increased during wetting process and decreases during drying processes. It can also been observed that the uncertainty of the safety factor of Case 1 is always the largest among the results of all three cases during the whole process of drying and wetting. After 12 hours of rainfall, the COV of safety factor is close to 20% for Case 1. While the COV of safety factor for Case 2 is only about 10%. As the drying function is used in the wetting cycle in Case 1, the water that can infiltrate into the slopes in Case 1 is much more than those slopes in Case 2 and Case 3. Therefore, the uncertainty of pore-water distribution in the slopes is largest and hence the uncertainty of the slope safety factor after raining is greatest in Case 1.

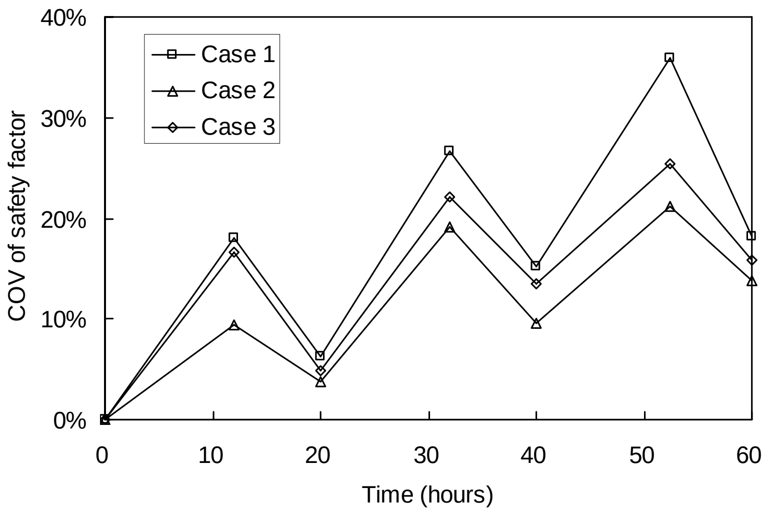

Fig.3 Variation of COV of safety factor with time

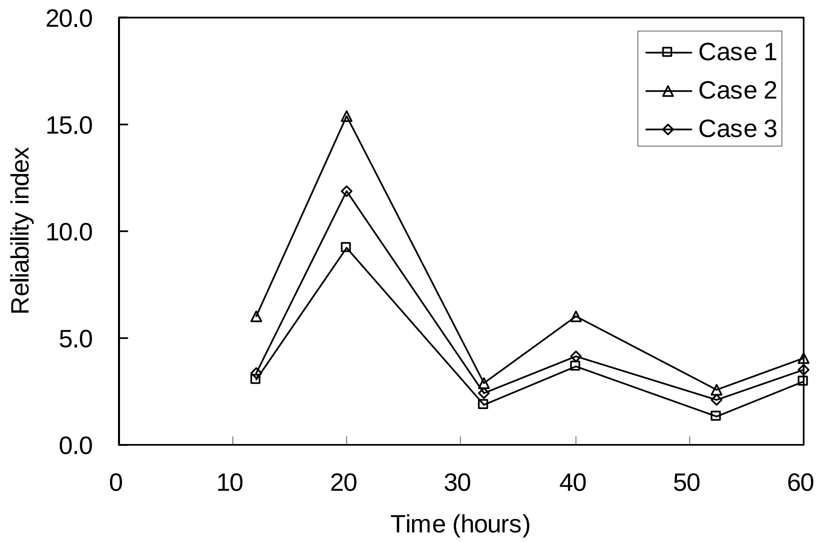

Fig.4 Variation of reliability index with time

In Fig.4, the change of reliability index with time is illustrated. The relibility index decreases with the precipitation and increaes in the drying process. After 60 hours of wetting and drying process, the reliability index of the slope is reduced to 2.95 for Case 1. But the reliability index of the slope is 4.07 for Case 2. Therefore, without considering soil hydraulic hysteresis, the reliability of the slope will be underestimated. It also shows that although the result of Case 3 is close to that of Case 2, it still underestimate the reliability of the slope especially during the initial stage of rainfall. It means the simplified assumption about soil hydraulic hysteresis using the average SWCC is not enough for a more valid evaluation of slope reliability during wetting and drying cycles.

4 CONCLUSIONS

In this paper, the effects of soil hydraulic hysteresis on reliability of an unsaturated soil slope during cycles of precipitation and evaporation are investigated. The findings of the results can be summarized as follows:

1. For all the three cases with different assumptions about soil hydraulic hysteresis, the mean of the safety factor and the reliabilitty index of the slope decrease during the precipitation and increase during the evaporation. The COV of the safety factor increased during wetting processes and decreased during drying processes.
2. The variation of the mean and the COV of the safety factor with time are most significant in Case 1 where the drying hydraulic function is used in both drying and wetting process. The COV of the safety factor of Case 1 is the largest among the results of all three cases during the whole process of drying and wetting.
3. Without considering soil hydraulic hysteresis, the reliability of the slope will be underestimated.

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