

An EXCEL add-in implementation for collocation-based stochastic response surface method

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ABSTRACT: A general probabilistic method called collocation-based stochastic response surface method (CSRSM) was developed previously. It involves the propagation of input uncertainties through a computation model to arrive at a random output vector. It is assumed that the unknown random output can be expanded using a polynomial chaos basis with corresponding unknown coefficients. The unknown coefficients are evaluated using a collocation method because it has the important practical advantage of allowing existing deterministic numerical codes to be used as "black boxes". An EXCEL add-in is developed to produce the basis functions (multi-dimensional Hermite polynomials) without resorting to symbolic algebra practitioners. This is a major practical advantage that would bring realistic probabilistic analyses within reach of the practitioners. Full EXCEL implementation details are illustrated using a simple slope problem involving six input random variables. The results show the ease and successful implementation of the proposed EXCEL-based CSRSM.

1 INTRODUCTION

There are many probabilistic methods in the literature, such as Monte Carlo simulation-based methods, response surface methods and stochastic finite element methods. The current challenge to the geotechnical engineering profession is to develop powerful and yet user-friendly probabilistic methods for realistic practical problems. It is well recognized that Monte Carlo simulation-based methods are computationally expensive. Stochastic finite element methods require significant modification of the existing deterministic numerical code and are impossible to apply for most engineers with no access to the source code of their commercial softwares. An alternate strategy is to replace the expensive numerical model with an approximate but cheaper surrogate version, i.e. so called response surface. The response surface methods consist of selecting different values of input parameters, running the simulation model to get samples of response, and then fitting a closed-form model which acts as a surrogate to the actual input-output relationship. The new adopted surrogate model or metamodel can be used to predict the response and carry out the uncertainty analysis. The conventional response surface (Schueller et al., 1989; Raymond and Douglas, 1995) employs a polynomial in terms of the input variables, based on regression analysis on data obtained from an appropriate design of experiments. However, conventional response surface may not always be a convergent series (Rebba, 2002). On the other hand, a stochastic response surface using polynomial chaos is known to be convergent (Ghanem and Spanos, 1991). The basic idea of collocation-based stochastic response surface method (CSRSM) is to reduce an unknown random output into polynomial chaos basis with corresponding coefficients and the unknown coefficients are evaluated using a collocation method (Tatang, 1995; Isukapalli, 1999; Huang, et al. 2007). It has the important practical advantage of allowing existing deterministic numerical codes to be used as "black boxes". The roots of the Hermite polynomial provide efficient collocation points to evaluate the coefficients in the stochastic response surface. Numerical examples (Huang, et al. 2007; Phoon & Huang 2007)

have proved that CSRSM can produce accurate estimates of the output CDF while requiring much fewer model simulations as compared to direct Monte Carlo simulation.

However, CSRSM has not been used more extensively. From a computation viewpoint, the approach requires two key steps: the first is computing multi-dimensional Hermite polynomials; the second is selecting collocation points. At present, multi-dimensional Hermite polynomials are mostly computed using symbolic algebra based on the differential definition of multi-dimensional Hermite polynomials. It is very difficult for most engineers to derive these polynomials as it requires significantly more mathematical effort than that required in design code/software applications (Ghamen, 1991). An original and efficient 2-term recurrence method for obtaining two dimensional Hermite polynomials of any order is proposed by Phoon and Huang (2007). Nevertheless, extending the recurrence method to $n > 2$ could be very complicated and difficult to be visualized at present. Isukapalli (1999) attempted to implement a web-based solution, but it is limited to $n < 5$ and limited to 2nd order or 3rd order Hermite polynomial, which may not be enough for some practical reliability problems in geotechnical engineering. Based on an ancient game, Sudret and Kiureghian (2000) constructed a creative model of “balls & boxes” which provided a less abstract interpretation. Further work is needed to bring this method within reach of practicing geotechnical engineers.

Practical computational details in CSRSM are the focus of this paper. In CSRSM, the deterministic response evaluation and stochastic analysis are decoupled. Hence, the selection of collocation points is independent of the physical problem. The main objective of this paper is to develop EXCEL add-in to produce multi-dimensional Hermite polynomials for practitioners without requiring extensive mathematical knowledge and programming ability. It is noted that the probabilistic analysis is easy to implement within EXCEL. Full EXCEL implementation details are given.

The advantages of the method as a bridge between stand-alone FEM packages and spreadsheet-based probabilistic analysis are demonstrated and discussed through a slope reliability problem involving six input random variables. The results show the ease and successful implementation of the proposed procedure for reliability analysis of a slope.

2 COLLOCATION-BASED STOCHASTIC RESPONSE SURFACE METHOD (CSRSM)

2.1 Multi-dimensional polynomials

The random model input parameters ($X_1, X_2, \dots, X_N$) representing load, soil, and/or geometrical parameters, can be correlated and non-Gaussian and can be expressed as functions of standard normal variables ($U_1, U_2, \dots, U_n$). The random output (can be a vector but assumed to be scalar for illustration) from a numerical code can be expressed as an implicit function of random input parameters, i.e. $Y = f(X_1, X_2, \dots, X_n)$. As a result, the output Y is usually non-Gaussian, following an unknown probability distribution function. If the output is a vector containing say m components ($Y_1, Y_2, \dots, Y_m$), the unknown to be computed is a m -dimensional joint probability distribution function. In the case of model inputs with more than one random component, model output can be represented by multi-dimensional Hermite polynomials:

$$Y = a_o + \sum_{i_1=1}^n a_{i_1} \Gamma_1(U_{i_1}) + \sum_{i_1=1}^n \sum_{i_2=1}^{i_1} a_{i_1 i_2} \Gamma_2(U_{i_1}, U_{i_2}) + \sum_{i_1=1}^n \sum_{i_2=1}^{i_1} \sum_{i_3=1}^{i_2} a_{i_1 i_2 i_3} \Gamma_3(U_{i_1}, U_{i_2}, U_{i_3}) + \dots \quad (1)$$

where Y is the output and $\Gamma_p(U_{i_1}, \dots, U_{i_p})$ are multi-dimensional Hermite polynomials of degree p given by:

$$\Gamma_p(U_{i_1}, \dots, U_{i_p}) = (-1)^p e^{\frac{1}{2}U^T U} \frac{\partial^p}{\partial U_{i_1} \dots \partial U_{i_p}} e^{-\frac{1}{2}U^T U} \quad (2)$$

where $\{U_{i_k}\}_{k=1}^p$ is a vector of independent standard normal variables, n is the number of standard normal random variables used to represent the uncertainty in the model inputs, and $a_0, a_{i_1}, a_{i_1 i_2}$, etc. are the coefficients to be estimated. $\Gamma_p(U_{i_1}, \dots, U_{i_p})$ are successive polynomial chaoses of their arguments. The polynomial chaoses of order greater than one have zero mean and polynomials of different order are orthogonal to each other. Details for calculating the polynomial chaos expansion can be found in the literature (Ghanem and Spanos, 1991). Two approximate second- and third-order forms are respectively given below (Isukapalli, 1999):

$$X \approx a_0 + \sum_{i=1}^n a_i U_i + \sum_{i=1}^n a_{ii} (U_i^2 - 1) + \sum_{i=1}^{n-1} \sum_{j>i}^n a_{ij} U_i U_j \quad (3a)$$

$$X \approx a_0 + \sum_{i=1}^n a_i U_i + \sum_{i=1}^n a_{ii} (U_i^2 - 1) + \sum_{i=1}^n a_{iii} (U_i^3 - 3U_i) + \sum_{i=1}^{n-1} \sum_{j>i}^n a_{ij} U_i U_j + \sum_{i=1}^n \sum_{j=1}^n a_{ijj} (U_i U_j^2 - U_i) + \sum_{i=1}^{n-2} \sum_{j>i}^{n-1} \sum_{k>j}^n a_{ijk} U_i U_j U_k \quad (3b)$$

For $n = 3$, Eqs. (3a) and (3b) produce the following expansions (indices for coefficients are re-labeled consecutively for convenience and clarity):

$$X \approx a_0 + a_1 U_1 + a_2 U_2 + a_3 U_3 + a_4 (U_1^2 - 1) + a_5 (U_2^2 - 1) + a_6 (U_3^2 - 1) + a_7 U_1 U_2 + a_8 U_1 U_3 + a_9 U_2 U_3 \quad (4a)$$

$$X \approx a_0 + a_1 U_1 + a_2 U_2 + a_3 U_3 + a_4 (U_1^2 - 1) + a_5 (U_2^2 - 1) + a_6 (U_3^2 - 1) + a_7 U_1 U_2 + a_8 U_1 U_3 + a_9 U_2 U_3 + a_{10} (U_1^3 - 3U_1) + a_{11} (U_2^3 - 3U_2) + a_{12} (U_3^3 - 3U_3) + a_{13} (U_1 U_2^2 - U_1) + a_{14} (U_1 U_3^2 - U_1) + a_{15} (U_2 U_1^2 - U_2) + a_{16} (U_2 U_3^2 - U_2) + a_{17} (U_3 U_1^2 - U_3) + a_{18} (U_3 U_2^2 - U_3) + a_{19} U_1 U_2 U_3 \quad (4b)$$

In general, the number of terms required respectively for the second-order (Eq. 4a) and third-order (Eq. 4b) expansions (Isukapalli, 1999) are given below (N denotes the number of terms and the subscript denotes the order of the expansion):

$$N_2 = 1 + 2n + \frac{n(n-1)}{2} \quad (5)$$

$$N_3 = 1 + 3n + \frac{3n(n-1)}{2} + \frac{n(n-1)(n-2)}{6}$$

For a fairly modest random dimension of $n = 6$, N_2 and N_3 are respectively equal to 28 and 84. Hence, fairly tedious algebraic expressions are incurred even at a third-order truncation. At present, multi-dimensional Hermite polynomials are mostly computed using symbolic algebra based on Eq. (2). EXCEL add-in is developed to produce these multi-dimensional Hermite polynomials. Details are presented in section 3.

2.2 CSRS algorithm

The uncertainties in random input parameters ($X_1, X_2, \dots, X_n$) are propagated to the output Y which is a deterministic function of inputs. The output Y is usually non-Gaussian, following an unknown probability distribution function. The stochastic collocation method as a probabilistic uncertainty propagation method can be applied in the analysis of physical systems in order to quantify the effects of random variation in the inputs on the predicted output of the simulation. The method can be summarized as follows:

1. Representation of the random inputs in terms of standard random normal variables by transformation (random variable input that can be transformed into standard random variables explicitly) or a series expansion, e.g., K-L expansion for random field input (Huang, et al., 2007). Non-Gaussian random vectors can be constructed by applying the appropriate cumulative distribution functions to each component of a suitably correlated Gaussian vector (Phoon, 2004) or Nataf transform (Sudret & Der Kiureghian, 2000).
2. Once the inputs are expressed as functions of the selected standard normal random variables, the output metrics can also be represented as functions of the same set of normal random variables. The minimum number of standard normal random variables needed to represent the inputs is defined as the “number of degrees of freedom” in input uncertainty. Since model outputs are deterministic functions of model inputs, they have the same number of degrees of freedom in uncertainty and can be expressed using the polynomial chaos series expansion given in Eq. (1). For a random dimension $n = 2$ and a third-order approximation ($p=3$), we have:

$$\begin{aligned} f(u_{i1}, u_{i2}) &= a_0 + a_1 u_{i1} + a_2 u_{i2} + a_3 (u_{i1}^2 - 1) + a_4 (u_{i2}^2 - 1) + a_5 u_{i1} u_{i2} + \\ &\quad a_6 (u_{i1}^3 - 3u_{i1}) + a_7 (u_{i2}^3 - 3u_{i2}) + a_8 (u_{i1} u_{i2}^2 - u_{i1}) + a_9 (u_{i2} u_{i1}^2 - u_{i2}) \\ &= a_0 + a_1 n_{i1} + a_2 n_{i2} + \dots + a_9 n_{i9} \end{aligned} \quad (6)$$

where (u_{i1}, u_{i2}) is one possible realization from a standard Gaussian vector (U_1, U_2) , $(1, n_{i1}, n_{i2}, \dots, n_{i9})$ are Hermite basis functions evaluated at (u_{i1}, u_{i2}) , and $(a_0, a_1, a_2, \dots, a_9)$ are unknown but constant numbers.

3. The unknown numbers can be determined using 10 different realizations. In matrix notation, we have:

$$Na = f \quad (7)$$

where N is a 10×10 matrix with i th row given by $(1, n_{i1}, n_{i2}, \dots, n_{i9})$, f is a 10×1 vector with i th component given by $f(u_{i1}, u_{i2})$ and a is a 10×1 vector containing unknown numbers $(a_0, a_1, a_2, \dots, a_9)'$. This is known as the stochastic collocation method. It is also possible to solve for the unknown coefficients using regression if there are more than 10 realizations.

$$N^T Na = N^T f \quad (8)$$

where N is a $r \times 10$ matrix and r is the number of realizations.

3 EXCEL ADD-IN FOR CSRSRM

3.1 EXCEL add-in implementation

This section discusses the development of an EXCEL Add-in for CSRSRM application. The add-in packages the main algorithm of CSRSRM in a convenient form for practitioners. Practitioners only need to enter a few key parameters describing the uncertainty information about the inputs and the output values from the physical model at collocation points.

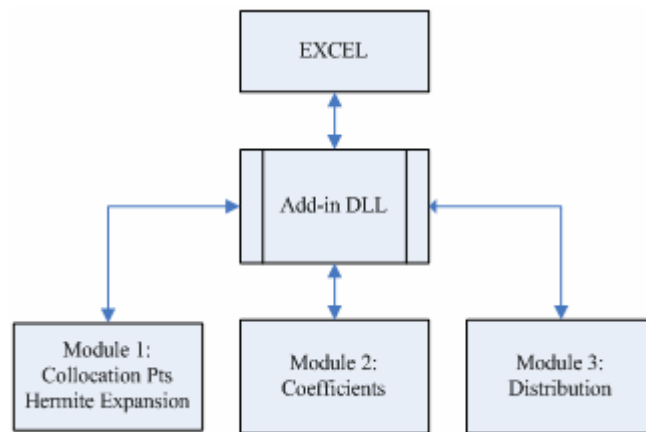


Fig.1 Framework of the add-in

The add-in DLL for the main program is developed in Visual Basic, which provides a friendly user interface. The lower level modules are developed in Visual C++. The structure of the software is shown in Fig.1. Data are passed between each module by text files. Flowchart of the software is presented in Fig.2.

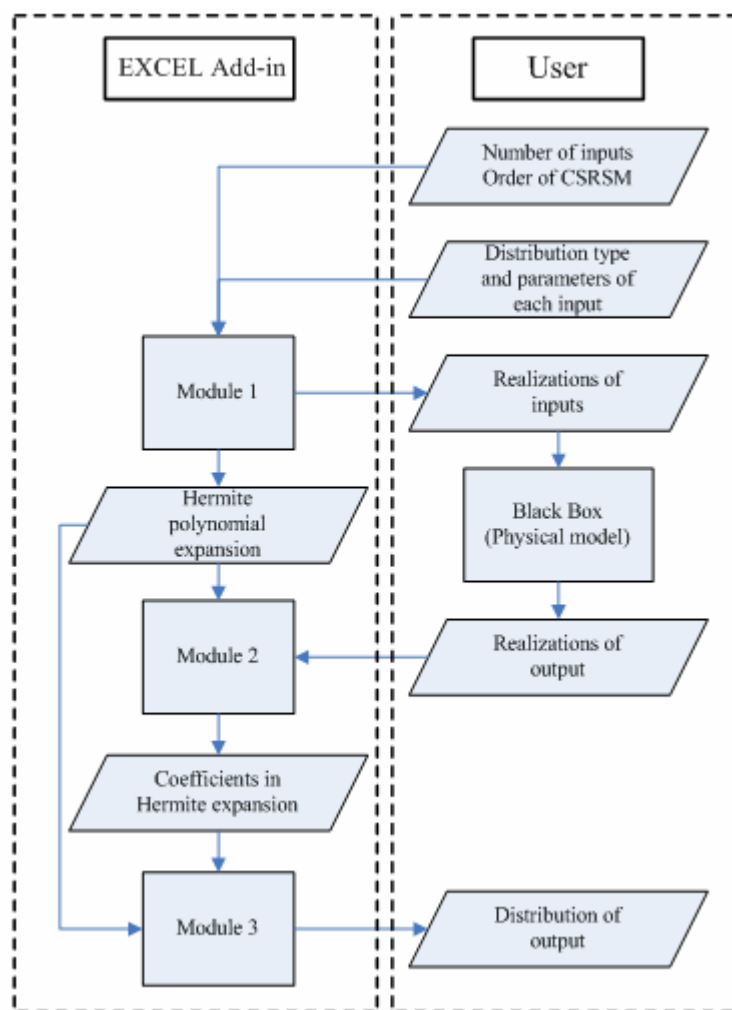


Fig.2 Flowchart

The add-in DLL is an EXCEL platform that allows users to invoke the three modules in a simple way. It mainly provides three functions: receiving data entered by users, calling other modules and displaying corresponding results in the EXCEL. The *Add-in Name* appears on the Excel Add-in window below when the add-in is installed.



The toolbar shown above includes four buttons. When the first three buttons are pressed, users would be asked to enter parameters if necessary, then the relevant module is called, and the results would be displayed in EXCEL. The fourth button presents an About Dialog of the software when pressed. The implementation of the three main modules will be discussed in detail in the following sections.

3.1.1 Module 1: Collocation Points and Hermite Expansion

In this module, three functions are achieved in sequence: 1) generating the value of inputs ($X_1, X_2, \dots, X_n$) at all collocation points; 2) deriving Hermite polynomial expansion; 3) giving the matrix **N** in Equation (7) by substituting all the collocation points into the expansion.

Because the polynomial chaos is the Hermite polynomial of the Gaussian variables, the collocation points are roots of the next higher order Hermite polynomial. For example, the first to fourth order Hermite polynomials are:

$$\begin{aligned} H_0(U) &= 1 \\ H_2(U) &= U \\ H_3(U) &= U^2 - 1 \\ H_4(U) &= U^3 - 3U \\ H_{k+1}(U) &= UH_k(U) - kH_{k-1}(U) \end{aligned} \quad (9)$$

The collocation points for the two dimension, second order response surface are combinations of the roots of the third order Hermite polynomial $(-\sqrt{3}, 0, +\sqrt{3})$. Hence, the number of collocation points is $3^2 = 9$. The corresponding physical inputs ($X_1, X_2, \dots, X_n$) of arbitrary distribution can be obtained from the collocation point u by employing the transformation shown in Table 1. It is possible to achieve relate the actual physical inputs ($X_1, X_2, \dots, X_n$) and the standard normal random variables ($U_1, U_2, \dots, U_n$) using one-dimensional Hermite polynomials as well (Phoon, 2004).

Table 1 Common transformations for random variables with different non-normal distributions

Distribution Type	Transformation
Uniform (a, b)	$a + (b - a) \left[\frac{1}{2} + \frac{1}{2} \operatorname{erf} \left(\frac{u}{\sqrt{2}} \right) \right]$
Normal (μ, σ)	$\mu + \sigma u$
Lognormal (μ, σ) *	$\exp(\mu + \sigma u)$
Gamma (a, b)	$ab \left(u \sqrt{\frac{1}{9a}} + 1 - \frac{1}{9a} \right)^3$
Exponential (λ)	$-\frac{1}{\lambda} \log \left[\frac{1}{2} + \frac{1}{2} \operatorname{erf} \left(\frac{u}{\sqrt{2}} \right) \right]$

* μ and σ are two parameters for the lognormal distribution, referring as the mean and standard deviation of the equivalent normal. Let μ_p and σ_p denote the actual mean and standard deviation of physical variables, then μ and σ are related to μ_p and σ_p by the following transformations:

$$\mu = \ln \left(\frac{\mu_p}{\sqrt{1 + \delta_p^2}} \right) \quad (10)$$

$$\sigma = \sqrt{\ln(1 + \delta_p^2)} \quad (11)$$

Where δ_p is the coefficient of variation:

$$\delta_p = \frac{\sigma_p}{\mu_p} \times 100\% \quad (12)$$

Then symbolic differentiation would be performed according to Eq. (2). Reader can refer to the theory of Polish notation and expression trees for detailed information (Drozdek, 2002). Finally, all the collocation points would be substituted into each term of the expansion, and the results would be saved as text files.

3.1.2 Module 2: Coefficients

In this module, coefficients of Hermite polynomial expansion would be estimated by solving Eq. (7): $Na = f$. Suppose the number of coefficients is m , and the number of collocation points which are chosen by user is n , then N is $n \times m$, a is $m \times 1$, and f is $n \times 1$. If m is equal to n and N is invertible, then the solution is given by $a = N^{-1}f$. However, N is not necessarily invertible and in most cases, n is bigger than m , so that the pseudoinverse method (or Moore-Penrose generalized inverse) is employed (Haville, 1997). If the pseudoinverse of N is denoted as N^+ , the solution can be presented as $a = N^+f$. According to the properties of pseudoinverse, solution $a = N^+f$ is the least squares solution of $Na = f$ and $\|f - Na\|$ attains its minimum value (Haville, 1997).

3.1.3 Module 3: Distribution

This module is designed to generate realizations of the output and then the cumulative distribution function of the output can be estimated. Once coefficients in polynomial chaos expansion are obtained, the output can be generated by Eq. (3) in which standard Gaussian random variables (mean=0 and variance=1) can easily be generated by random number generator and coefficients a are known.

3.2 Illustrative numerical example

Table 2 Probabilistic description of input random variables.

Variable	Description	Distribution	Statistics
H	Depth of soil above bedrock	Uniform	[2,8] m
$h = H \times U$	Height of water table	U is uniform	[0, 1]
ϕ	Effective stress friction angle	Lognormal	mean = 35° cov = 8%
β	Slope inclination	Lognormal	mean = 20° cov = 5%
γ	Moist unit weight of soil	*	*

γ_{sat}	Saturated unit weight of soil	**	**
γ_w	Unit weight of water	Deterministic	9.81 kN/m ³

* $\gamma = \gamma_w (G_s + 0.2e)/(1+e)$ (assume degree of saturation = 20% for “moist”).

** $\gamma_{sat} = \gamma_w (G_s + e)/(1+e)$ (degree of saturation = 100%).

Assume specific gravity of solids = G_s = uniformly distributed [2.5, 2.7] and void ratio = e = uniformly distributed [0.3, 0.6].

3.2.1 Example description

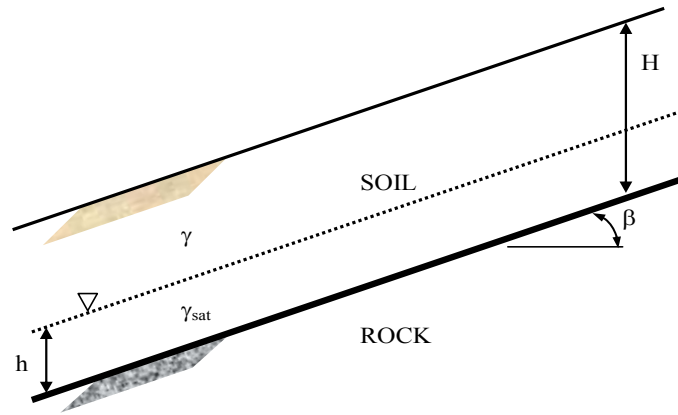


Fig.3 Infinite slope problem.

The example considered is an infinite slope problem (Fig.3). The factor of safety is given by:

$$FS = \frac{c + [\gamma(H - h) + h(\gamma_{sat} - \gamma_w)] \cos^2 \beta \tan \phi}{[\gamma(H - h) + h\gamma_{sat}] \sin \beta \cos \beta} \quad (13)$$

Assuming effective cohesion, $c = 0$, the performance function (G) is given by:

$$G = FS - 1 = \frac{[\gamma(H - h) + h(\gamma_{sat} - \gamma_w)] \cos \beta \tan \phi}{[\gamma(H - h) + h\gamma_{sat}] \sin \beta} - 1 \quad (14)$$

where H , h , γ , γ_{sat} , ϕ and β are input random variables, whose physical description and statistics are described in Table 2. Note that the moist and saturated soil unit weights are not independent, because they are related to the specific gravity of the soil solids (G_s) and the void ratio (e). The uncertainties in γ and γ_{sat} are produced by modeling G_s and e as two independent uniform random variables. In summary, this example involves six independent random variables: H , U , ϕ , β , e , and G_s (see Table 2). The covs are reasonable with reference to extensive statistics compiled by Phoon & Kulhawy (1999a; 1999b).

3.2.2 Using the EXCEL add-in

In this example, the output is the performance function (G) which is expressed by six independent input random variables. The following tutorial demonstrates how to use the EXCEL add-in for CSFEM analysis:

- (1) First, to input the number of variables and the order of polynomial chaos expansion, click on the “collo” button. A dialog box appears (as shown below).

Parameters

Number of uncertain inputs in the model
(up to 10 supported):

Order of Polynomial Chaos Expansion
(1-5 supported):

- (2) Then, proceed with “Next” button to state distribution type of each variable and assign the parameters in another dialog box (as shown below).

Parameters

Inputs	Distribution Type	Parameter A	Parameter B
Input1	Uniform (a, b)	2	8
Input2	Uniform (a, b)	0	1
Input3	Lognormal (mu, sigma)	3.552158	0.0798724
Input4	Lognormal (mu, sigma)	2.994484	0.0499688
Input5	Uniform (a, b)	2.5	2.7
Input6	Uniform (a, b)	0.3	0.6

Note that here the parameters σ and μ of Lognormal Distribution are not mean and standard deviation of the lognormal variables but those of normal equivalents (see Eqs. 10, 11 and 12).

- (3) Go to next step by clicking on “Next” button. A series of calculation will be performed: Values of each input variables at all the collocation points will be listed below. Then users can choose the collocation points in which outputs are desired and put the results in the corresponding cells for output Y . If a collocation point is not chosen, the corresponding cell is blank.
- (4) Then, to estimate the coefficients, just go ahead with “Coeff” button. The result will also be shown in EXCEL. In order to estimate the coefficients correctly, the number of chosen points must be no less than the number of terms of the Hermite polynomial expansion; otherwise an error message would appear.

	A	B	C	D	E	F	G	H	I
1		X1	X2	X3	X4	X5	X6	Enter Y in this row: Coefficients:	
2	Col Pt1	7.748329	0.5	34.88853	19.97505	2.6	0.45	0.433822558	0.454937
3	Col Pt2	2.251671	0.5	34.88853	19.97505	2.6	0.45	0.433822558	0
4	Col Pt3	5	0.958055	34.88853	19.97505	2.6	0.45	0.040307219	-0.245183
5	Col Pt4	7.748329	0.958055	34.88853	19.97505	2.6	0.45	0.040307219	0.153339
6	Col Pt5	2.251671	0.958055	34.88853	19.97505	2.6	0.45	0.040307219	-0.080015
7	Col Pt6	5	0.041945	34.88853	19.97505	2.6	0.45	0.875314382	0.008504
8	Col Pt7	7.748329	0.041945	34.88853	19.97505	2.6	0.45	0.875314382	-0.015768
9	Col Pt8	2.251671	0.041945	34.88853	19.97505	2.6	0.45	0.875314382	0
10	Col Pt9	5	0.5	40.06508	19.97505	2.6	0.45	0.729351596	0
11	Col Pt10	7.748329	0.5	40.06508	19.97505	2.6	0.45	0.729351596	0
12	Col Pt11	2.251671	0.5	40.06508	19.97505	2.6	0.45	0.729351596	0
13	Col Pt12	5	0.958055	40.06508	19.97505	2.6	0.45	0.254727748	0
14	Col Pt13	7.748329	0.958055	40.06508	19.97505	2.6	0.45	0.254727748	0
15	Col Pt14	2.251671	0.958055	40.06508	19.97505	2.6	0.45	0.254727748	0.008175
16	Col Pt15	5	0.041945	40.06508	19.97505	2.6	0.45	1.261840492	-0.025463

value of inputs

leave blank if not chosen

- (5) Once the coefficients are determined, the Hermite polynomial expansion is also determined. By pressing the “Distrb” button, we simulate realizations of the output randomly using Eq. (3). A dialog box is provided to specify the number of realizations. Simulation time required for Eq. (3) is quite reasonable. It takes approximately 10 seconds to generate 10,000 realizations on a common computer (AMD 2600+, 512M).
- (6) After the whole process is completed, the program will show where the results have been saved, and users can utilize other statistical tools to analyze and display the results as shown in Fig.4.

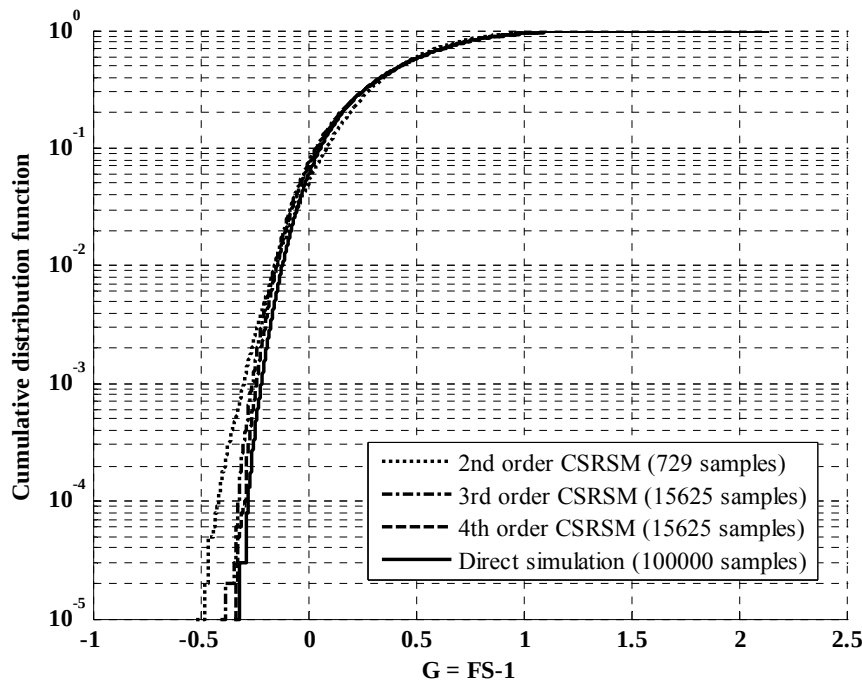


Fig.4 Cumulative distribution function of performance function

4 CONCLUSIONS

In CSRSM, the deterministic response evaluation and stochastic analysis are decoupled. Hence, the selection of collocation points is independent of the physical problem. The EXCEL add-in is developed to produce multi-dimensional Hermite polynomials for practitioners without requiring

extensive mathematical knowledge and programming ability. It stands as a platform between stand-alone FEM packages and spreadsheet-based probabilistic analysis. A slope reliability problem involving six input random variables shows the ease and successful implementation of the proposed procedure for reliability analysis of a slope.

Since there are many possible combinations of the roots, for the higher dimensional and higher order polynomial chaos, the number of available collocation points is always greater than the number of collocation points needed. Therefore, a selection of the appropriate collocation points from the large number of potential candidates is a practical question. Model inputs can be correlated non-normal random variables or non-normal random fields. Further work including collocation selection and dealing with correlated non-normal inputs is in progress.

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