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Uncertainty analysis on remoulded undrained shear strength of marine clay

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ABSTRACT: Remoulded undrained shear strength $s_{u\text{-}r}$ is a soil property associated with disturbed soil. This parameter is used to indicate the evolution of the post-failure behaviour of seafloor soil, estimate the rheological properties of debris flows and the drag forces on seafloor structures exerted by a submarine slide, and calculate the penetration resistance of skirt walls. This work discusses the influence of uncertainty on the estimate of empirical relationships between $s_{u\text{-}r}$ and liquidity index I_L as a potential predictor of the remoulded soil strength. Special emphasis is made on assessing the uncertainty inherent to $s_{u\text{-}r}$, when the definition of the empirical models is updated from a global to a regional data set. For this purpose two approaches are followed: a parameterisation in the 'log space' (log-Linear Model, LLM) by transforming least square exponential fitting to log-linear expression, and a parameterisation in the 'physical space' using Bayesian Non-Linear Regression (BNLR). The updating of the trend models conditioned on the two available data sets is performed using simple Bayesian principles. Results show contrasting trends obtained by the two approaches. A thorough discussion is included based on the different hypothesis considered for the estimate of each model.

1 INTRODUCTION

There is normally limited data for soil investigation, especially for the soil exploration in deep water environments. As a consequence, the uncertainty involved in the estimation of soil properties is a primary concern in the definition of design parameters. Previous work by Nadim and Locat (2005) and by Nadim et al. (2005) has identified potential sources of uncertainty during the determination of soil parameters for offshore sites. However, it is usual to find that empirical correlations based on in-situ or laboratory methods, traditionally used to estimate soil parameters, do not consider the effect of the parameters uncertainty.

The remoulded undrained shear strength $s_{u\text{-}r}$ is used for the analysis of mobility of debris in a retrogressive slide, where it is considered to give a lower bound estimate of strength during the first part of the debris run-out process (Kvalstad et al., 2005). Additionally, it has been observed that the drag forces on the structures on the seafloor are proportional to the $s_{u\text{-}r}$ value assigned to the sliding mass (Demars, 1978). Furthermore, $s_{u\text{-}r}$ is considered a key parameter for estimation of the force and deformation of the seafloor structures exerted by a submarine slide (Demars, 1978), and it has been used to calculate the penetration resistance of the skirt walls (Dendani, 2003). Due to the relevance of this parameter, this work discusses the influence of uncertainty on the estimate of empirical relationships between $s_{u\text{-}r}$ and liquidity index I_L. The liquidity index can be established easily in the laboratory and it exhibits strong correlation with $s_{u\text{-}r}$ (Yang et al., 2006).

Special emphasis is made on assessing the uncertainty inherent to $s_{u\text{-}r}$, when the definition of the empirical models is updated from a global to a regional data set. For this purpose two approaches were followed: a parameterisation in the 'log space' (log-Linear Model, LLM) by transforming the least square exponential fitting to log-linear expression, and a parameterisation on the 'physical space' using Bayesian Non-Linear Regression (BNLR). A comparison between trends is thus possible when translating the trend models between the log and the physical spaces. In addition, the updating of the trend models conditioned on a global and a more recent regional data set was performed using simple Bayesian principles. This allowed for the quantification of uncertainties at both the physical level (strength estimates) and at the parameter level (model parameters). Updating has been used in parameter estimation for reducing uncertainty of prediction from empirical correlations (Zhang, et al., 2004), in hydrology (Kaheil, et al., 2006), in inverse problem for steady-state ground water flow and heat transport (Jiang and Woodhury, 2006), in environmental risk assessment (Bates, et al., 2003), in characterization of geological anomaly (Halim and Tang, 1990), and in the probabilistic calibration of soil models (Medina-Cetina, 2006), among others.

Results of the estimates of the empirical relationships show contrasting trends as obtained by the LLM and the BNLR approaches. A thorough discussion is included based on the different hypothesis considered for the estimate of each model.

2 STUDY DATA SETS

The main motivation for initiating the collaboration that resulted in this paper is the analysis of parameters of the Storegga Slide. The Storegga Slide occurred about 8200 years ago (Haflidason et al. 2005). Its slide mass involved approximately 3000 km^3, affecting an area of 90,000 km^2 (Kvalstad et al., 2005 b). The Storegga initial slide comprised over 1000 km^3 of soil and had a maximum run-out distance of 410 km for the debris flow, and possibly 800 km for the turbidite (Issler et al., 2005). A number of geotechnical tests have been performed on samples from the boreholes in this area, populating a detailed database of geotechnical, geological and geophysical parameters, and allowing for the investigation of parameter relationships within this geological setting (Yang et al., 2006).

Due to the importance of the contribution of marine deposits to the overall Storegga slide, especially to the long run-out distance of this slide and potential impact to pipelines, attention was paid to the remoulded undrained shear strength $s_{u\text{-}r}$ of the marine sediments in the Storegga Slide region. Kvalstad et al. (2005) indicated that the run-out distance of Storegga slide is strongly dependent on the remoulded strength of the material in the slip surface. The shear strength can be reduced further after intrusion of water or remoulded soft material from the previous side block.

In this work, $s_{u\text{-}r}$ is characterised as a random variable, so that uncertainties involved in its assessment can be retrieved from available data. A database has been built for the assessment of soil design parameters from index measurements in clay at NGI (NGI, 2002). The global data set for marine clay in this study came from this database, which includes 12 sites from the Norwegian Sea, the North Sea, the Atlantic Ocean, the west coast of Ireland, offshore Angola, offshore Nigeria and the Gulf of Mexico. $s_{u\text{-}r}$ was measured by fall cone tests, and the liquidity index was calculated as

$$I_L = \frac{w_i - w_p}{w_l - w_p} \tag{1}$$

where, w_i represents the water content, w_p represents the plastic limit, and w_l represents the liquid limit.

3 METHOD OF ANALYSIS

3.1 Estimates in the log-space: Log-Linear Model LLM

Zhang, et al. (2004) discussed the parameterisation of an empirical soil model where the dependent variable and parameters of the model were assumed to be random variables. This is one possible way for assessing the influence of the uncertainty in the model predictions, where both the calibration error and the model variation itself are conditioned on observations (data).

The assumptions considered in the LLM approach followed in the reference paper are:

- The distribution of the dependent variable *y* is assumed as normal
- The correlation trend line between y and *x* is assumed to be linear
- The scatter about the trend line is assumed to be constant at all values of *x*

- The weight contribution of each of the data points is assumed to be equal.

It is worth noticing that in practice, the distribution of y may not be normal and the scatter about the trend line may not be constant, as it will be discussed below. Consider also that a log-normal variable can be normalized by transforming the original domain (referred here as 'physical' domain) to the log domain. This is actually a very common practice among Engineers and Scientists made for ease of data interpretation.

In this paper, we aim at finding an empirical correlation between a soil physical index such as IL and $s_{u\text{-}r}$. Of special interest is to measure the effect of parameterising the empirical model first using an existing global data set from different sources of information collected from different sites, and then updating it using a regional data set collected from the Storegga Slide area. The purpose of the model updating is founded in the need for considering previous experience generated from other sites, instead of ignoring previous sources of information, or just to contrast the case where the two sets are added together.

In our case, a linear relationship of the type

$$y = a + bx \tag{2}$$

was found suitable for estimating the dependent variable $s_{u\text{-}r}$ when it is translated from the physical space to the log space. Thus, let us define y as the transformed variable of $s_{u\text{-}r}$, a and b as the corresponding regression coefficients, and x as the independent variable representing IL. The parameter statistics for calculating the log-linear parameters are introduced below.

A measure of the calibration error about the regression line (which is assumed to be a constant) can be defined as (Walpole, et al., 1998):

$$\sigma^2 = \frac{\sum_{i=1}^{n} (y_i - y_{ei})^2}{n-2} \tag{3}$$

where y_i is a data point (in the log space), y_{ei} is the corresponding log-linear estimate, and n represents the number of data points (samples).

The variance of the mean at a specific point $x = x_0$ is defined as (Walpole, et al., 1998):

$$\sigma_{m0}^2 = \sigma^2 \left(\frac{1}{n} + \frac{(x_0 - x_m)^2}{(n-1)s_x^2}\right) \tag{4}$$

where σ is the calibration error given in equation (3), n is the number of samples, x_0 is the point where σ_{m0}^2 is estimated, x_m is the mean of the independent variable x, and s_x^2 is defined as:

$$s_x^2 = \left(\sum (x_i - x_m)^2\right) / n \tag{5}$$

where x_i represents the location of the data points.

The variance of y_e estimated at x_0, σ_0^2 is assumed to be equal to the variance of the mean at x_0 plus the variance about the mean (Benjamin and Cornell, 1970):

$$\sigma_0^2 = \sigma^2 + \sigma_{m0}^2 \tag{6}$$

In this way, it is possible to estimate the mean y_{e0} (trend model) and variance σ_0^2 (around the model) of y_e for each x_0 for both the global (index 1) and regional data sets (index 2). Moreover, if it is assumed that residual between the data y_i and the model y_{e0} for the global and regional data sets follow a Gaussian behaviour, the updated estimate of the mean and standard deviation of the remoulded undrained shear strength in the log-space can be calculated using the Bayesian paradigm as (Ang and Tang 1975, Walpole, et al., 1998):

$$u''_0 = \frac{y_{e01}(\sigma_{02})^2 + y_{e02}(\sigma_{01})^2}{(\sigma_{01})^2 + (\sigma_{02})^2} \tag{7}$$

and

$$\sigma''_0 = \sqrt{\frac{(\sigma_{01})^2(\sigma_{02})^2}{(\sigma_{01})^2 + (\sigma_{02})^2}} \tag{8}$$

respectively, where, y_{e01} and y_{e02} are the trend means from the two data sets, and $\sigma_{01}{}^2$ and $\sigma_{02}{}^2$ are their corresponding variances.

Finally, by transforming the estimates of the dependent variable from the log to the physical space, the mean and variance of the dependent variable at any x_0 can be obtained as (Walpole, et al., 1998):

$$\mu_{s_{u-r}0} = e^{\left(y_{e0} + \frac{\sigma_0^2}{2}\right)} \tag{9}$$

$$\sigma_{s_{u-r}0} = \sqrt{e^{\left(2y_{e0} + \sigma_0{}^2\right)}.(e^{\sigma_0{}^2} - 1)} \tag{10}$$

3.2 Estimates in the physical-space: Bayesian Non-Linear Regression BNLR

In the physical space it is proposed an exponential relationship between the remoulded shear strength and the liquidity index of the form:

$$s_{u-r} = B_0 \exp(B_1 I_l) \tag{11}$$

Note that a biunique relationship between the trends of y and s_{u-r} is expected since they can be transformed from both spaces and converge to the same trend. In our case, the parameterisation of this model is performed using the Bayesian paradigm, which is defined as (Papoulis, 1991):

$$\pi(\theta \mid d_{obs}) = \frac{f(d_{obs} \mid \theta)\pi(\theta)}{\int f(d_{obs} \mid \theta)\pi(\theta)d\theta} = \frac{f(d_{obs} \mid \theta, g(\theta))\pi(\theta)}{\int f(d_{obs} \mid \theta, g(\theta))\pi(\theta)d\theta} \tag{12}$$

where the prior $\pi(\theta)$ introduces the a-priori state of information associated to the a set of parameters θ, which in our case is defined as the vector of the model parameters $\theta = \{B_0, B_1, \sigma\}$, where σ is a measure of the calibration error. The likelihood $f(d_{obs} \mid \theta)$ represents the a-priori state of information associated to the potential of the parameters θ to match the observations $d_{obS} = y_i$ or to help to match them if they are embedded into a predictive model $g(\theta) = y$, which in our case is the empirical model given by equation 11. The posterior $\pi(\theta \mid d_{obs})$ is thus the joint probability function between the a-priori states of information associated to both the prior and the likelihood.

For the parameterisation of the exponential model introduced in equation 11, it is assumed that the coefficients $\theta = \{B_0, B_1, \sigma\}$ follow vague priors, and that the likelihood $f(d_{obs} \mid \theta)$ follows a Gaussian-type behaviour. Under these assumptions, the posterior assumes the form:

$$\pi\left(\theta \mid d_{obs}\right)=\pi\left(B_0,B_1,\sigma \mid I_L,s_{u-r}\right)\propto \frac{1}{\sigma^{n+1}}\exp\left(-\frac{1}{2\sigma^2}\sum_1^n\left(s_{u-ri}-B_0\exp\left(B_1 I_{Li}\right)\right)\right) \tag{13}$$

For estimating first and second moments of the posterior, or for estimating marginal statistics for each parameter included in θ, it is required to integrate the posterior with respect to the parameters domain. This integral can become cumbersome when the number of parameters spans in a high dimensional space. Fortunately the integral can be solved numerically. This solution yields a full description of the uncertainty associated to the model parameters θ conditioned on the available data, which contrasts with the LSLR approach, where only best estimates are retrieved.

An efficient way to solve the posterior integral is using Markov Chain Monte Carlo MCMC (Robert and Casella, 2004). A useful property of the MCMC is that it converges to the target joint density as the sample integration grows. For this work, the decision rule that determines which samples are 'accepted' or 'rejected' is the Metropolis-Hastings (MH) criteria. Under these premises, the posterior integration at the MH 'state' of the chain *s + 1* iteration is obtained by sampling a candidate point **Y** from a proposal distribution $q\left(\cdot \mid \hat{\theta}_s\right)$, where the candidate point **Y** is accepted or rejected as the next step of the chain with probability given by:

$$\alpha\left(\hat{\theta}_s,d_{obs}\right)=\min\left\{1,\frac{\pi\left(Y \mid d_{obs}\right)q\left(\hat{\theta}_s \mid Y\right)}{\pi\left(\hat{\theta}_s \mid d_{obs}\right)q\left(Y \mid \hat{\theta}_s\right)}\right\} \tag{14}$$

For the MCMC sampling the distribution of interest $f\left(\cdot \mid d_{obs}\right)$ appears as a ratio, so that the constant of proportionality cancels out. Additionally, the evaluation of the posterior requires discarding the first iterations called the burn-in points, before it reaches the stationary condition from which the statistical inferences are generated.

4 CASE STUDY

In the following sections, the influence of 'updating' empirical relationships when going from a global data set to the most recent influence of a regional data set is explored. For this purpose, two comparative approaches are developed, one where the updating is performed in the log space, and another one where the updating is performed in the physical space. A comparison between the two updating procedures is discussed below.

4.1 Updating in the log space

Based on the NGI database (NGI, 2002), an empirical exponential relationship was obtained representing the global data set (*n* = 138 samples, equation 15). Estimates of the correlation coefficient and calibration error were 0.924 and 10.77 respectively. Equation 16 introduces the resulting trend model obtained in the log space from equation 15:

$$s_{u-rg}=163.7\exp(-3.915 I_L) \tag{15}$$

$$\ln(s_{u-rg})=\ln(163.7)-3.915 I_L \tag{16}$$

where s_{u-rg} is the remoulded undrained shear strength from the global data in kPa and I_L is the liquidity index.

A second regression equation was obtained using the Storegga data set or regional data (*n* = 50 samples, equation 17). The correlation coefficient and the calibration error were 0.664 and 46.32 respectively. Equation 18 introduces the resulting trend model obtained in the log space from equation 17:

$$s_{u-rr} = 210.3\exp(-4.143 I_L) \tag{17}$$

$$\ln(s_{u-rr}) = \ln(210.3) - 4.143 I_L \tag{18}$$

where s_{u-rr} is the remoulded undrained shear strength from the regional data in kPa and I_L is the liquidity index.

The remoulded undrained shear strength is updated using the information from global and regional data sets in the log space, and using equations 7 and 8. The updated mean and standard deviation of the remoulded undrained shear strength is shown in Fig.1. The same curves plotted in the physical space are presented in Fig.2. From these figures it is observed that the updated trend model approximates the model obtained from the regional data set. The updated standard deviation on the other hand, shows smaller values than any of the two individual models obtained before, with a relative small gain with respect to the trend model obtained from the global data set.

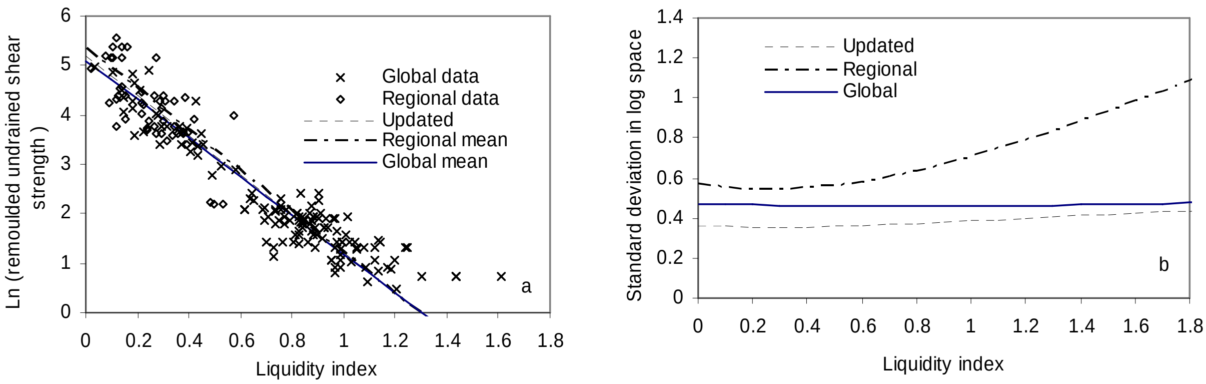


Fig.1 a) Mean of the remoulded undrained shear strength from global, regional data sets and the updated analysis, b) Standard deviation for global , regional data sets and updated analysis.

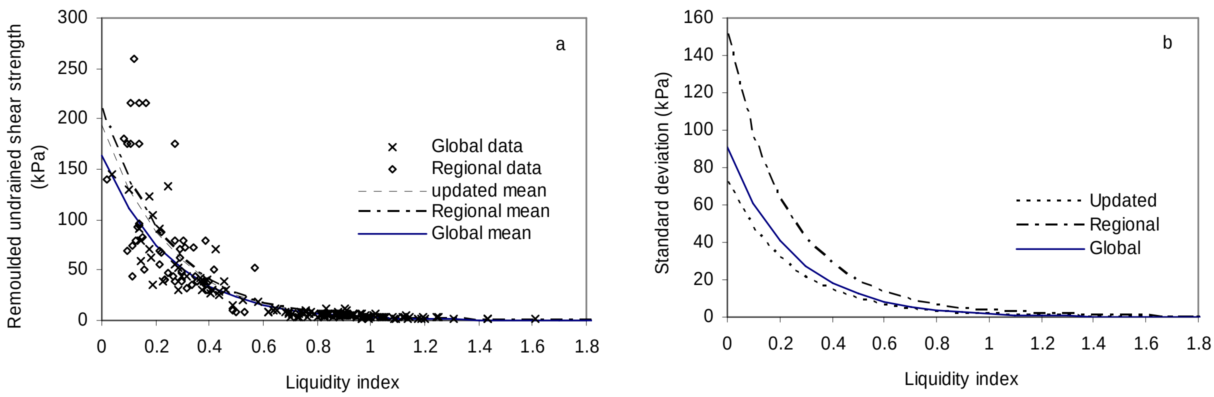
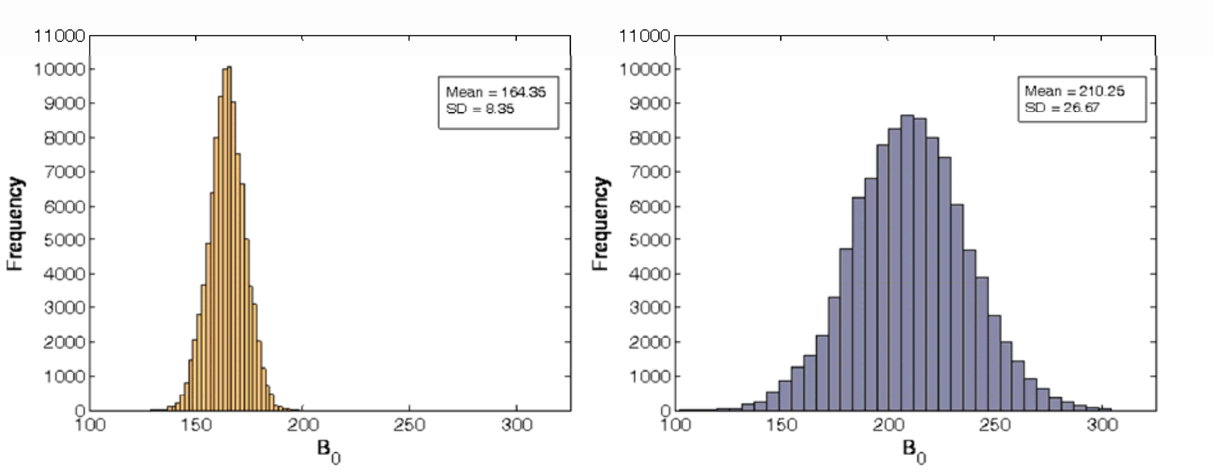


Fig.2 a) Remoulded undrained shear strength from global, regional data sets and the updated analysis, b) Standard deviation for global , regional data sets and updated analysis.

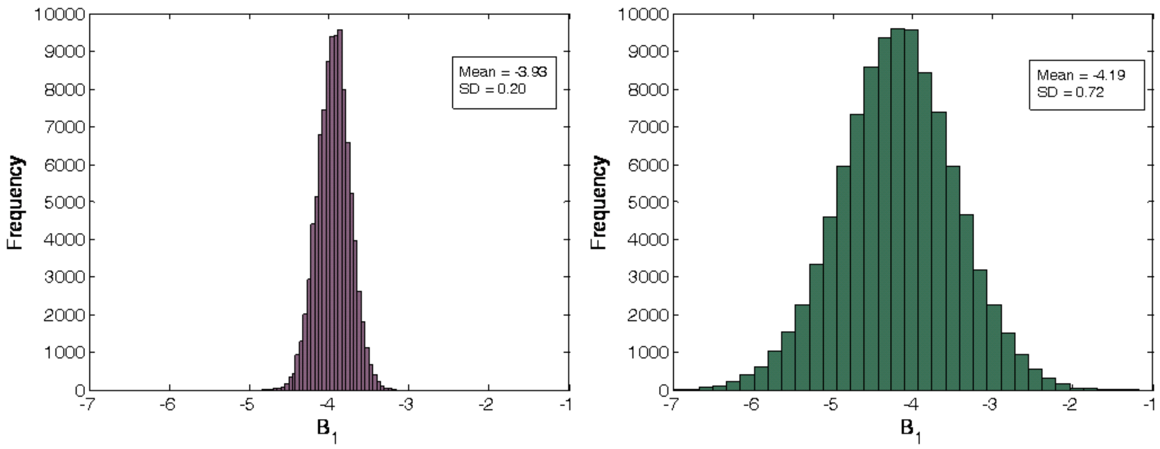
4.2 Updating in the physical space using Bayesian Non-Linear Regression BNLR.

The solution of the numerical integration of the posterior was reached after taking 300,000 samples. The transition between the 'burn in' phase and the 'stationary' phase was estimated at the location of the 200,000th sample. Frequency histograms from the stationary phase for each of the model parameters are presented in Fig.3, 4 and 5, where both results from the global and regional data sets are included.



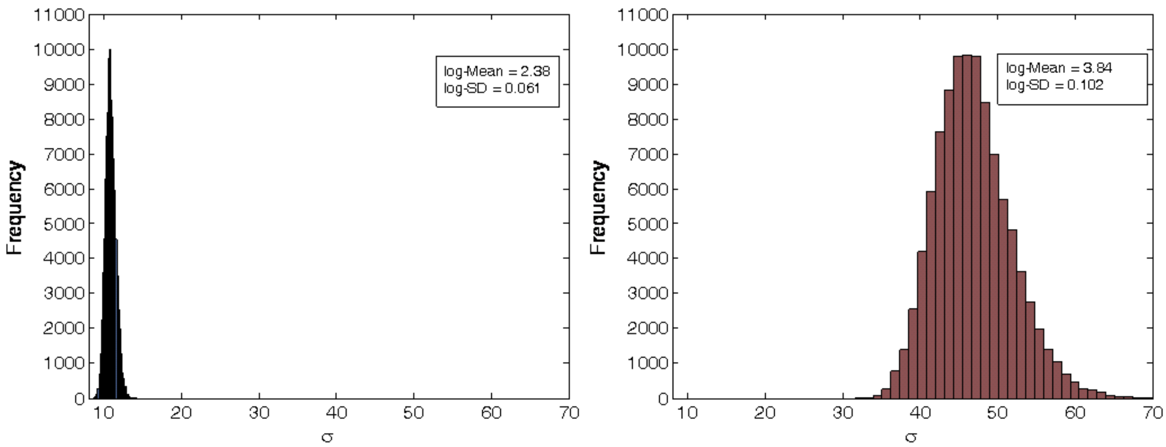
(a) (b)

Fig.3 Marginal distributions of B_0 using the global (a) and regional data sets (b).



(a) (b)

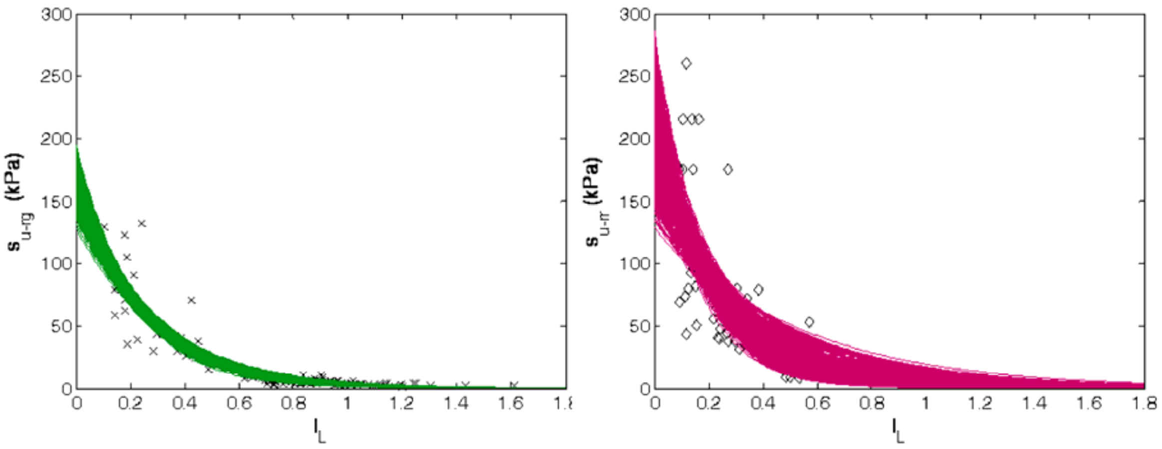
Fig.4 Marginal distributions of B_1 using the global (a) and regional data sets (b).



(a) (b)

Fig.5 Marginal distributions of σ using the global (a) and regional data sets (b).

These figures show the capability of the Bayesian approach to obtain a full probability description of the trend model parameters. Quantitatively, it is observed that the means of each distribution coincide with those obtained by the *LLM* approach, which proves the consistency of the method. Also, despite the natural shift on the means of each model when going from the global to the regional data set, the uncertainty of each parameter is significantly more in the case of the regional parameters. Based on these results, it is thus possible to simulate likely realisations of the trend mean by sampling each marginal probability distribution. Results of the simulations are presented in Fig.6 for both the global (a) and regional (b) data sets.



(a) (b)

Fig.6 Trend model simulations (10,000) based on the marginal probability density distributions of the trend model parameters for the global (a) and regional (b) data sets.

By looking at the marginal distributions of the s_{u-r} trends generated from the realizations showed in Fig.5 at different values of I_L for both data sets, it is observed that these evolve from a Gaussian-type at $I_L = 0.0$ to an exponential-type at $I_L = 1.8$, which means that the updating of the trend model cannot be done in the physical space since it would be a violation to the hypothesis required by the Bayesian formulation. In this regard, it is worth noticing that by following the BNLR approach it is possible to confirm this non-Gaussian effect, whereas in the case of the LLM it is 'assumed' that it follows a Gaussian behaviour

An interesting feature found in the empirical distributions of the model parameters, is that the coefficients B_0 and B_1 follow a Gaussian distribution, while the measure of the calibration error σ follows a log-normal distribution. The corresponding statistics are showed in a box on each histogram plot of Fig.3, 4 and 5 respectively. Since the distribution of the three parameters follows a Gaussian-type behaviour, it is thus possible to update the model information from the global to the regional data sets using a Bayesian formulation similar to the equations 7 and 8, but at the parameters' level. This results in the estimate of updated probability density functions for each parameter, as shown in Fig.7.

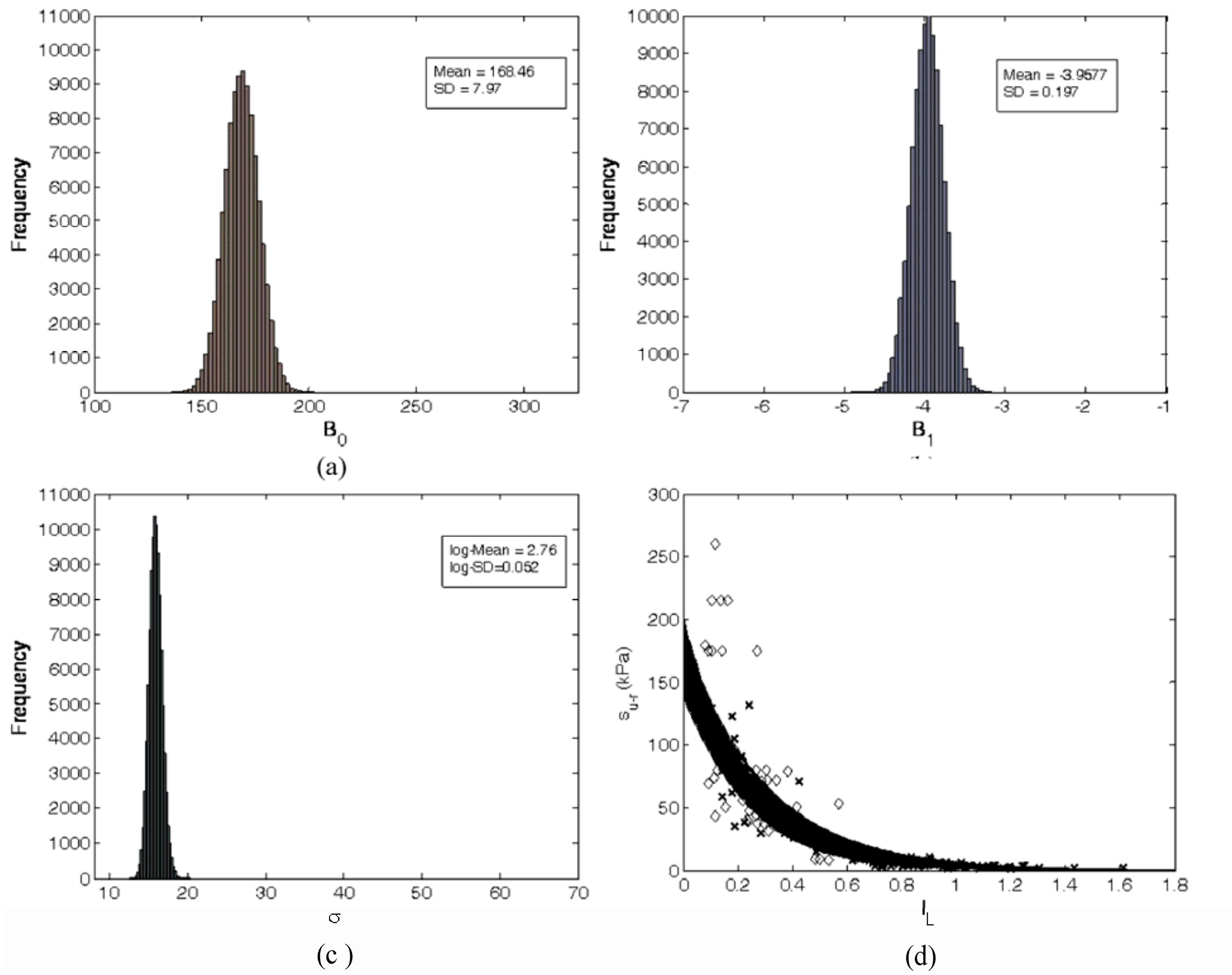


Fig.7 Updated marginal distributions of parameters B_0 (a), B_1 (b), σ (c), and trend model simulations (10,000) based on these updated parameter distributions (global and regional data plotted in the background as reference only).

From the statistics presented on Fig.7a-c (shown in text boxes), it is observed that parameters distributions tend to exhibit a similar behaviour to those by the global data set (Fig.3a, 4a and 5a), with a small reduction on the uncertainty for all three. Additionally, Fig.7d shows likely simulations of the trend model resulting from the parameters update. Marginal distributions of the s_{u-r} trend along the I_L domain were similar to those observed in Fig.6 for the global and regional data sets. Statistics from these marginal distributions are presented in Fig.8, which presents a comparative analysis between the mean and standard deviation of the trend models for the global, regional and the updated distributions along the I_L domain.

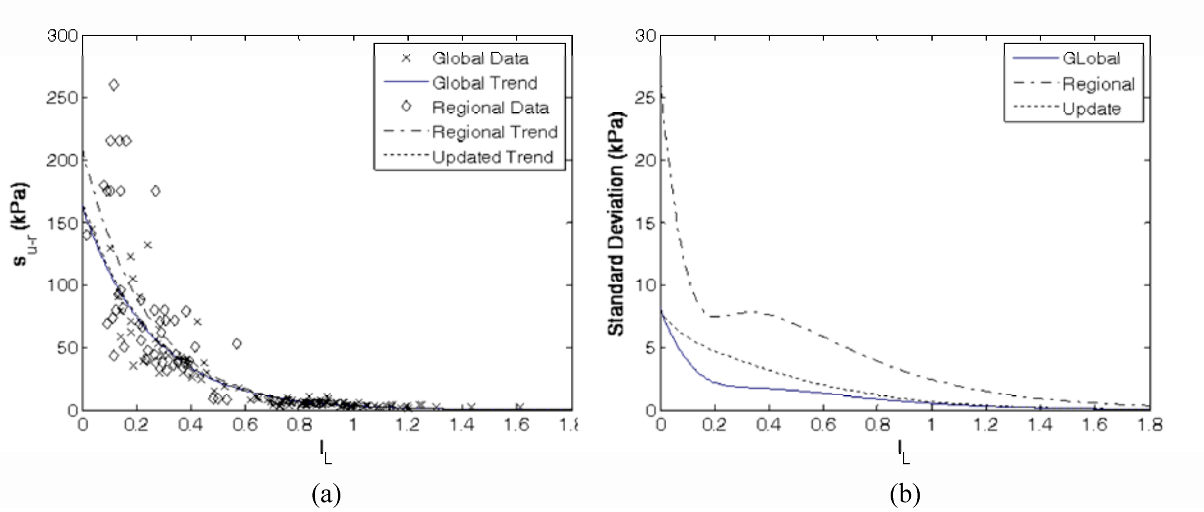


Fig.8 Mean (a) and standard deviation (b) of the trend models based on probability distributions of the model parameters estimated from the global data set.

This figure shows that the updated trend model tends to be the same as the one obtained from the global data set (a). The uncertainty associated to estimate of each of these means (b) shows that the updated variation of the trend mean lies between the standard deviation of the means obtained with the global and regional data sets. Since it is evident that the calibration error is non-constant in the physical domain as it appears to be in the log-domain, it is not suggested to add the uncertainties according to the equation 6. Nevertheless, it is possible to compare the uncertainty associated to the estimates of the trend model with that calculated in the log space as showed in Fig.9. This figure shows that the update of the uncertainty associated to the trend model is smaller than those estimates obtained from the global and regional data sets.

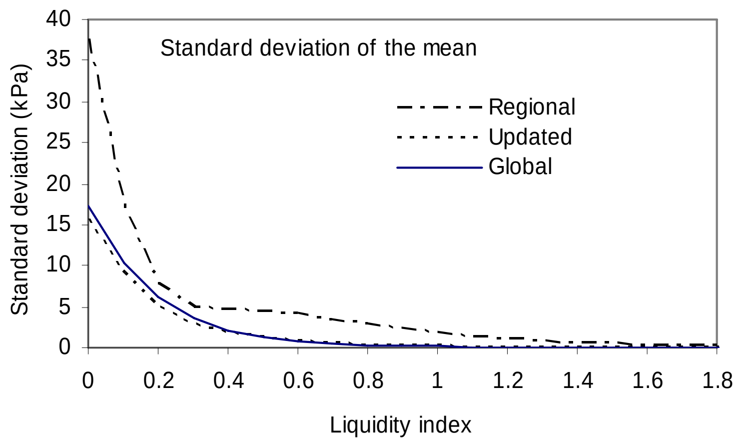


Fig.9 Standard deviation of the mean from log space analysis

5 CONCLUSION AND DISCUSSION

The remoulded undrained shear strength from a global data set was updated by a regional data set from Storegga Slide region. Two approaches were applied in the case study. The classical log-linear model indicated that the updated mean is close to the mean value from the regional data set, and the updated standard deviation is less than those from both of the data sets. The analysis in the physical space by updating the parameters in the exponential relationship showed that the updated mean is close to the mean value from the global data set, and the updated standard deviation is less than that from the regional data set and higher than that from the global data set. The first method in log space, simple to use in practice, and the results appear to be rational as the updated estimates are close to the site-specific estimates. The second method is based on a more formal application of the Bayesian methodology by updating the parameters in the physical space. However, at first glance the analysis results do not seem to be consistent as the updated estimates are very close to the global estimates for low values of liquidity index. This is most likely due to the unusual situation where the dispersion in the site-specific data is greater than the dispersion in the global data.

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