Shallow Foundation Reliability Design

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ABSTRACT: The increasing awareness that soils are materials that, even in a lithologic homogeneity, show pronounced variability of their physico-mechanical properties, has given a remarkable increase to the development of probabilistic computational models in geotechnics. Significant reflexes are present also in geotechnical regulations, in which (see Eurocode 7 for example) the research of characteristic values and the use of partial coefficients prove to be a significant step forward compared to the use of Global safety factors. In this note the problems of shallow foundations are taken into account, as far as the bearing capacity and settlements are concerned, remarking some of the most significant studies carried out in the last years on probabilistic basis.
The outcoming frame is that there is still a lot to study as far as the statistic structure of input data is concerned, the models to use the verification in real magnitude of the results obtained from calculations.

1 INTRODUCTION

The more and more detailed knowledge of the mechanical behavior of soils and the interaction soil-structure have allowed to develop considerably the calculation procedures for the achievement of well defined safety levels in geotechnic field.

There has already been a significant step forward from the classical Global safety factors, including indiscriminately all the possible uncertainties in the procedures of physical and mechanical characterization of soils and calculation to the partial ones able to better calibrate the entity of the penalizations of the Resistances and the amplifications of the Actions.

A further step to the more aware safety has been for some time identified in the direction of the evaluation of the so called Reliability Index, connected to the probability that the unfavorable event, for example that the overcoming of the limit bearing capacity takes place.

In order to obtain the previously wished results, even in classical situations as planning a foundation for geotechnical purposes, it is necessary to carry out a quantitatively and qualitatively significant characterization of soils and equally a calculation model .

In this note a series of aspects will be pointed out connected to planning in terms of reliability as far as the bearing capacity and settlements of shallow foundations are concerned, reporting and commenting some results obtained from different Authors.

2 SAFETY FACTORS IN GEOTECHNICS

In the recent past, in many regulations and codes, the safety of geotechnical works has been entrusted to an approach called WSD (Working Stress Design) based on the so called Global safety factor expressed as the ratio between the Resistances (indicated generally as R or C) and the Actions (indicated generally with Q or D), expressed both in terms of forces, moments or stresses considering obviously all the possible kinematics in the examined case.

Normally this ratio was based, even if not appositely specified, on actions and resistances calculated through the mean values of the relating input parameters

The understanding that, because of numerous uncertainties in play (between which the intrinsic variability of soils), those input parameters were not unambiguous but characterized by many times significant variability and consequently by specific frequency distributions has made understand that

Resistances and Actions couldn't be expressed by an univocal value but even by specific frequency distributions.

It is possible therefore to find the case in which, even being in presence of a very high deterministic safety factor, even higher than the limit fixed by the law, it is possible to notice a significant superimposition of the above mentioned frequency distributions individuating thus a finite probability that the Action overcomes the Resistance.

A further complication is represented by the circumstance that in the classical definition of the Safety Factor intended as ratio some forces, moments or stresses (and this happens very frequently in the case of verification of retaining walls) could be considered either as additional contribute to Resistance or as decrement to the Action with numerically different results (De Mello 1989, Li et al., 1993, Rethati, 1988).

Consequently Codes and regulations have been developed for which the resistances are evaluated through characteristic values (even on statistical basis) of the input parameters penalized with appropriate "partial safety factors", while the Actions are analogously amplified. This is the logic that the Eurocode 7 and the Italian T.U. follow, with small differences.

Operating in this way it is possible to take into account in a certain way of the variability of the parameters of resistance penalizing them and at the same time amplifying the actions, arriving at a final State limit approach (ULS).

A certain subjectivity remains in the chose of the characteristic values when, and this is the most frequent case, the input data are not sufficient for a characterization following a statistical approach.

A more comprehensive and complex approach in the evaluation of safety is given by probabilistic methods, that are based on the evaluation of the reliability through the so called Reliability Index β through which it is possible to reach the knowledge of the probability connected to the unfavorable event considered.

Before better specifying the meaning of β, that will be illustrated in the next paragraph it is appropriate to remind what it is meant with the term Reliability especially in the geotechnical field. For this purpose is it possible to mention Harr (1987) that affirms: "Reliability can be defined simply as the probability of success "(of a structure on soil or rock).

A more precise definition is the probabilistic assessment of the likelihood of the adequate performance of a system for a specified period of time under proposed operating conditions. The acceptance of a Reliability level must be viewed within the context of possible costs, risks and associated social benefits.

Nevertheless, to this clear definition corresponds a series of problems that will be pointed out in the following paragraphs.

3 SIGNIFICANCE AND VALIDATION OF MODELS

When we consider how varied and uncertain the problems concerned with soils are and the great number of variables involved in the issue, it seems often quite hard to work out a mathematical model which could actually represent the real situation by corresponding to the soil model.

Any model (Geymonat and Giorello 1980) that might be defined effective both from a theoretical and a technical point of view, should be a "restricted picture" or "simplified image" of reality, as in the model construction some characteristics of the real situation must necessarily be left aside, either regarding the soil or the calculation.(Aversa 1997).

Moreover, it must be said that our model (in our case the geotechnical one) is valid only insofar it can solve the original problems it was meant for.

An important distinction must be made between understanding and prediction .Although a more complex model might not lead to better prediction, it could give the designer a better understanding of it (Elms 2001).

For example, the model for bearing capacity calculation, with reference to foundations, is normally different from the one which is generally used to measure the stresses of the soil on the structure (though the Winkler type models are too simple, nonetheless these can still be considered adequate for the second scope indicated).

It would seem that several models can be applied to the same situation, such as, for instance, surface foundations, but out of the many approaches and workable models, the best and most competitive theory is that which gives the closest representation of reality, given the same degree of complexity. A model should be selected and chosen, provided it fulfils such a requirement: this is why the validation phase, when data are considered and compared, is considered of paramount importance in the theory development.

Early scientists in the past decades had already acknowledged that models were to be primarily evaluated in relation to their correspondence with reality; in fact, they used the following procedure:

- Data analysis for a particular type of soil or problem
- Creation of an "ad hoc" model to explain the analyzed phenomenon
- Model validation for a limited number of cases, say for analogous soils or problems.

Such a procedure can be defined as "empirical" or "inductive": it was also used by the early scientists in Galileo's age, later developed by the Positivists in 18th century. At the beginning of 20th c. the neopositivist movement contributed further to the theory and some thinkers in Vienna (Carnap, Reichenbach, Neurath among them) promoted the development of a new philosophy of science called epistemology, derived from a need to endow philosophy with new scientific criteria.

This was due to a radical change in the perception of life and world, brought forward by Einstein theories and quantum physics.

Neopositivist philosophers had a strong faith in rational thinking and in scientific progress; that is why they pursued "scientific truth" as a criterion to validate any theory: a model was to correspond to "real facts" and to direct observation of reality.

These scientists were proposing the three-phase procedure described above, which is an example of the "inductive method" Thus for them the correspondence between previsions and observation is the watershed separating a truly scientific theory from a non-scientific one. Any theory is valid, provided it can be verified.

Quoting Reichenbach (1930) we say: "The inductive principle determines the truth of scientific theories. If we exclude this principle, we deprive science of its power to state either the truth or the falseness in any theory ".

And yet, we can detect some problems in such a procedure. Firstly, the collected data though consistent, are necessarily restricted in number and only concerned with some defined cases; these might differ one another and so data would have an intrinsic measurement bias. Besides, even in the best model construction, only the most relevant parameters can be considered, as it is impossible to take into account the countless effects of a phenomenon.

On the contrary, we aim at constructing a model which could constitute a "general rule", i.e. one able to give outcomes for an endless number of initial situations and even for all those values which might have been left aside in the analysis.

To be defined correct and appropriate, a model must ensure a theorically endless number of previsions.

In order to fill the gap between the finite data, for the parameters considered and the theorically endless outcomes of the rule, we have nothing else available but the man – devised inductive method, whose hypotheses can never be proved as regards to their intrinsic truth. Then, we must conclude that the principle of the inductive method does not rest on a rational basis, as it is not possible to derive infinite results from a finite number of data.

Similarly, also the assumption that these models can be fully and satisfactorily tested is unfounded.

Indeed, experience drawn from direct observation, once compared with mathematical models, tells us that while it is quite impossible to rationally prove the truth, given an infinite content of an assumption, it is possible to have for any assumption a logical proof of its being false.

In other words, while we need endless trials for an assumption to be said as true, that very same assumption will be judged as false, should even a single evidence turn out as negative.

In his work about logical principles in scientific discoveries (1934), Popper starts from this logical asymmetry between truth and falseness to motivate his criticism towards traditional scientific procedures, which were following (and still do at times, nowadays) the neopositivist dogmas ; hence the need for him to work out new paradigms.

Popper argues that in comparison with factual data any theory tends to be marked with a negative criticism, as the comparison aims at showing the theory as false, instead of true. Even in case a model is successfully proved, after first trials, its validity is being reinforced and strengthened, but never fully confirmed.

There is not such of a scientific truth considered as certain: simply, any theory that has not yet been evaluated as false, will be temporarily considered as true until it is ousted by a new one.

In his epistemological research, Popper underlines another consequence which stems from the inconsistency of the inductive method: for which, he argues, scientists should not "induce" theories from facts; rather, when working out theories and models, they should simply derive just "deductions". Starting from either one or more hypotheses, only their direct logical consequences will be considered as true, and procedures will not be infringed by gaps in logical sequencing.

But the question is: what is, then, the role that factual data do play in a scientific deductive theory?

Facts do play a very important and crucial role: Popper clearly states in his criticism that though direct observation is often decisive in failing a theory as false, nonetheless it plays a fundamental role also in its original function. Indeed, in scientific research, scientists have an active role in the process of direct observation: each observed situation is preceded by problems, enquiries, hypotheses; in a word, it is largely a matter of selection and choice.

During one of his lectures (1949) Popper suggested that if we asked some students to attend an experiment and take note of the results, they would immediately ask in turn what to pay more attention to.

Similarly to those students, a scientist's mind should act as a lighthouse, that enlightens that part of factual knowledge that interests him most in relation to his/her own hypothesis: his mind should not simply be a container where every sensing perception is stored and from which a theory will be later drawn out.

The relationship between direct observation and models that, as we have already stated, is fundamental in scientific enquiry, was not Popper's concern only. It was later developed in detail by other philosophers and particularly by Popper's pupil and scholar Joseph Agassi, who shared with him the idea of "theory falsification". He devoted to the quality assessment of the practical controls that are necessary to define a theory as truly scientific. (1964).These are the criteria for effective controls:

- The result should not be dependent in any way, on the case under scrutiny or on the hypothesis itself (independence)
- The controls should be easy to access to (accessibility)
- It should be possible to apply them in all cases (universality)
- The more abstract they are, the more powerful as reinforcement (abstraction)
- They should give impressive results having deep impact (depth)

Imre Lakatos agrees on this point, when he warns scientists not to worry too much about "strenuously defending" their theories ; he suggests that they should have instead these theories further and further tested, going through more and more careful controls, as this is the condition for real advancement and progress.

His philosophy goes some steps further than Popper's, when he states that "falsification" is not an issue to be debated within theories themselves but across the boundaries of alternative research programs (1971), i.e. sets of subsequent theories developed from original ideational cores.

So, any anomalous event that might occur during observation, is coded as a proof of falsification against those research programs which are not able to provide with always "new" outcomes. A consequence is the reinforcement of alternative programs, which can be crucial in the long run and take to scientific revolutions.

As we have seen by laying out, though in short, the theories of the greatest epistemologists of our age, the inductive method has been ousted by a deductive method introduced by Popper, together with the notion of theory falsification.

These ideas were further developed by Agassi and Lakatos, who shifted stress from the defence of one's own theories to the main goal of scientific enquiry, which is progress.

Though quite different, Feyerabend's stance (1970) is remarkable and worthy some lines here, as he does not aim at setting objective criteria for scientific progress; he simply states that there can be more than a single way or type of control to evaluate real progress in science.

In his views, a scientist is like a child, who continuously shifts his attention onto new patterns and reacts to new stimuli. Hence, theories and controls are not one way related; even rational thinking might not fit to some cases just as there is no point in comparing two works of art or two great artists belonging to different schools.

Advocating a sort of “creative anarchy”, Feyerabend aims at a dynamic and creative interpretation of real facts, while Popper, Agassi and Lakatos aimed at working out defined reference schemes to evaluate either scientific objectivity or falsification.

Finally we can say that geotechnics has moved far from the questions so briefly discussed in this paper. This was due to some difficulties linked to the characteristics of the natural materials concerned. The process that goes on an increasing complexity of proposed models on the basis of real data must be encouraged, with probabilistic approaches constituting indeed a remarkable evolution because it helps to acquire greater safety levels. At any rate, it has to be taken into account that real data are the product of uncertain and variable conditions which cannot be easily described; hence we must argue that either validation or falsification evidence have to rest on purely statistical bases.

4 VARIABILITY AND UNCERTAINTY IN GEOTECHNICS

Four major sources of uncertainties can be identified in geotechnics and consequently in foundation design.

-The first source is related to intrinsic variability of physico-mechanical properties of soils due to not constant mineralogical composition, variable grain size distribution and stress history.

-The second is related to the limited number of tests and samples required for the evaluation of interest geotechnical properties.

It is fundamental however to put into evidence that whatever the test modalities may be (in situ and/or in laboratory) the sampled volume is, also for time and cost reasons, always a very small percentage of the volume effectively interested by significant tensional variations in relation to the examined problem (significative volume).

-The third is related to the procedures of in situ and in laboratory tests. For laboratory tests disturbances in sampling and preparing specimens are very important, and also measurement errors.

For in situ tests problems arise from using transformation models from measurements to desired parameters to be defined. For example deriving undrained shear strength from CPT tests, Relative density from SPT tests and so on.

-The fourth source of uncertainty results from the difference between the actual behaviour of soil and the design model. This source of uncertainty is mainly due to the necessary simplifications of model respect to effective reality.

After all it is possible to deal with Aleatory Uncertainty and Epistemic Uncertainty, in the following so defined (Baecher and Christian 2003):

Aleatory Uncertainty represents the natural randomness of a soil characteristic. It is also called inherent variability. It cannot be reduced.

Epistemic Uncertainty represents the uncertainty due to lack of knowledge of a soil characteristic. It includes measurement uncertainties due to measurements, to limited data disposable, to models. It can be reduced to a certain extent, for example increasing quality and quantity of measures.

In practical terms, for an efficient analysis with reliability methods let us consider that laboratory and in situ tests are performed in statistically meaningful number and that the sampling strategies are efficient.

Let us consider also that geotechnical parameters are spatially distributed variables.

If we assume that a data set is made up of measurements related to the depth (then the measured variable is considered as one dimensional spatial variable) such values k(z) can be divided into two components (eq.1): a deterministic trend (spatial mean) t(z) and residual values (random fluctuations about the trend) w(z):

$$k(z) = t(z) + w(z) \tag{1}$$

There are many possible trends for the spatial mean over the depth; it can be constant, linearly variable or it can follow a high order mathematical law. Moreover, also random part w(z) can be constant or variable respect to depth.

In order to study the variability structure of the measure k(x) it is necessary to focus the attention on the residuals. These latter values shall be isolated by the trend and analyzed by looking for their spatial distribution provided that they have a zero mean value and a standard deviation σ to be calculated.

Also important for probabilistic analyses is the evaluation of scale of fluctuation (δ) as a concise indicator of the spatial extent of a strongly correlated domain.

For determination of it various curve-fitting and statistical techniques are available in geotechnical literature (De Groot and Baecher 1993, Fenton 1999).

Also useful is the knowledge of Probability density function of variables (geotechnical parameters) and possible correlations between them.

Finally it must be highlighted that as previous steps in the analysis of data sets are necessary:

-To detect outliers by means of opportune procedures (Cherubini et al 1986);

-To detect homogeneous strata by "ad hoc" methods (Wickremesinghe and Campanella 1991, Gill 1970).

Further consideration on the topic that could even be useful would take a lot of space. For some deepening it is possible to make reference to Cherubini (1997) where it is possible to find useful considerations and tables concerning the variability of cohesive and cohesionless soils. Further exhaustive analyses and numerous data could be found in Phoon and Kulhawy (1999 a, b).

5 RELIABILITY INDEX EVALUATION

The Reliability index is generally so defined:

$$\beta = \frac{\mu_M}{\sigma_M} = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2 - 2r_{RQ}\sigma_R\sigma_Q}} \tag{2}$$

Where

M = R-Q = Safety Margin
R = Resistance (also defined as Capacity C)
Q = Loading (also defined as Demand D)
μ = Mean value
σ = Standard deviation
r = Correlation coefficient

or

$$\beta = \frac{E[F]-1}{\sigma_F} \tag{3}$$

Where F is the classical Safety factor. Beta is related to the probability of failure according to the distribution of M.If M is normally distributed or has a triangular distribution the relation is unique.

In the case of lognormal distribution of Performance Function (when β is expressed in function of F and F is lognormally distributed) the probability associated to a value of β is function of the coefficient of variation of F.

Because the assumption of normal distribution is conservative, it is reasonable generally to assume a normal distribution in absence of further information.

The solutions techniques to obtain β are several, characterized by advantages and disadvantages, and are reported in the text of Baecher and Christian (2003) and here summarized:

-The First Order Second Moment (FOSM) method, using the first terms of a Taylor series expansion in the Performance Function to estimate the expected value and the variance of Performance Function.

-The Second Order Second Moment (SOSM) method, using the terms of Taylor series up the second order. This method not always offer significant improvements on accuracy and is not widely used in geotechnics.

The Point Estimate Method (PEM) in the different versions proposed by Rosenblueth (1975, 1981) Harr (1989) and Zhou and Nowak (1988) obtains moments of the Performance function by evaluating it at a set of specifically chosen points.

-The Hasofer-Lind (1974) method also indicated as FORM representing an improvement of FOSM method based on a geometric interpretation of β as a measure of the distance in a dimensionless space between the peak of the multivariate distribution of the uncertain parameters and a function defining the failure condition. Low and Tang (1997) proposed a very efficient procedure to be solved by means of spreadsheets and mathematical computer codes.

The term SORM is applied to second order geometrical reliability analysis.

-The Monte Carlo simulation for which a large number of sets of randomly generated values for uncertain parameters are created to compute Performance function mean and standard deviation. This method does not give insight into the relative contributions of uncertain parameters, like previous methods.

-Stochastic Finite Element Methods. The geotechnical literature contains few descriptions of applications of this method of analysis. A possible application consists in a deterministic computational approach with uncertain properties having perfect correlations from point to point. Further approaches are described in Baecher and Christian (2003).

6 BEARING CAPACITY OF SHALLOW FOUNDATIONS

In this paragraph are reported and synthetically commented the results of some studies carried out from different authors for the evaluation of the bearing capacity using a probabilistic approach. The first are referred to essentially parametric studies, instead the last ones consider, even in different ways, real situations in all or in part.

El Meligy and Rackwitz (1999) hypothesize two different cases relating to isolated foundations, one on cohesive and the other one on cohesionless soil, c and φ varying respectively from 50 to 400 kPa and from 25° to 40°. In addition to c and φ they consider as variables even the dimensions of the foundation and their depth respect to the ground level, the values of eo and Gs as well as the loads imposed on the foundation divided in dead loads and live loads, using normal probability distributions and coefficients of variation inferred from literature.

The authors lead a sensitivity analysis making use of the code COMREL (using FORM and SORM methods).

The conclusions drawn from an analysis that tends to evaluate the reliability through the β index and the use of partial safety factors opportunely evaluated are:

- It is found that c and φ, strength variables, are the most important variables, while among loading variables eo and live loads evidence significant importance;
- For the case of cohesionless soil the normalized sensitivities are affected by the value of φ while they are not affected by the value of c in the case of cohesive soil;
- Consequently the Authors state that the use of constant partial safety factors in case of cohesionless soil may result in varying reliability level ; they recommend for the high COV of soil properties to have partial safety factors as function of the variability of c and φ.

It has to be pointed out the circumstance that for the calibration of the partial coefficients the Authors assume as characteristic values the mean values.

Cherubini (2000) has analyzed, using the procedure proposed by Low and Tang (1997), a shallow strip foundation 1.5 m wide and located at -1.2 m respect to the ground level.

The expression used for the calculation of the bearing capacity has been the classical trinomial one based on bearing capacity factors Nc, Nq, Nγ, performing the calculation in terms of effective stress.

The input data have been calibrated on the physico-mechanical characteristics of a cohesive soil characterized by γ=17.3 kN/m3, a mean value of cohesion equal to 14.4 kPa (with COV=10-20%) and a mean value of friction angle equal to 20° (with COV=20-30%).

These coefficients of variation have been assumed consistently with the data reported in literature for clays, and in the same way has been done for the fluctuation scale, assumed variable between 1 and 2m.

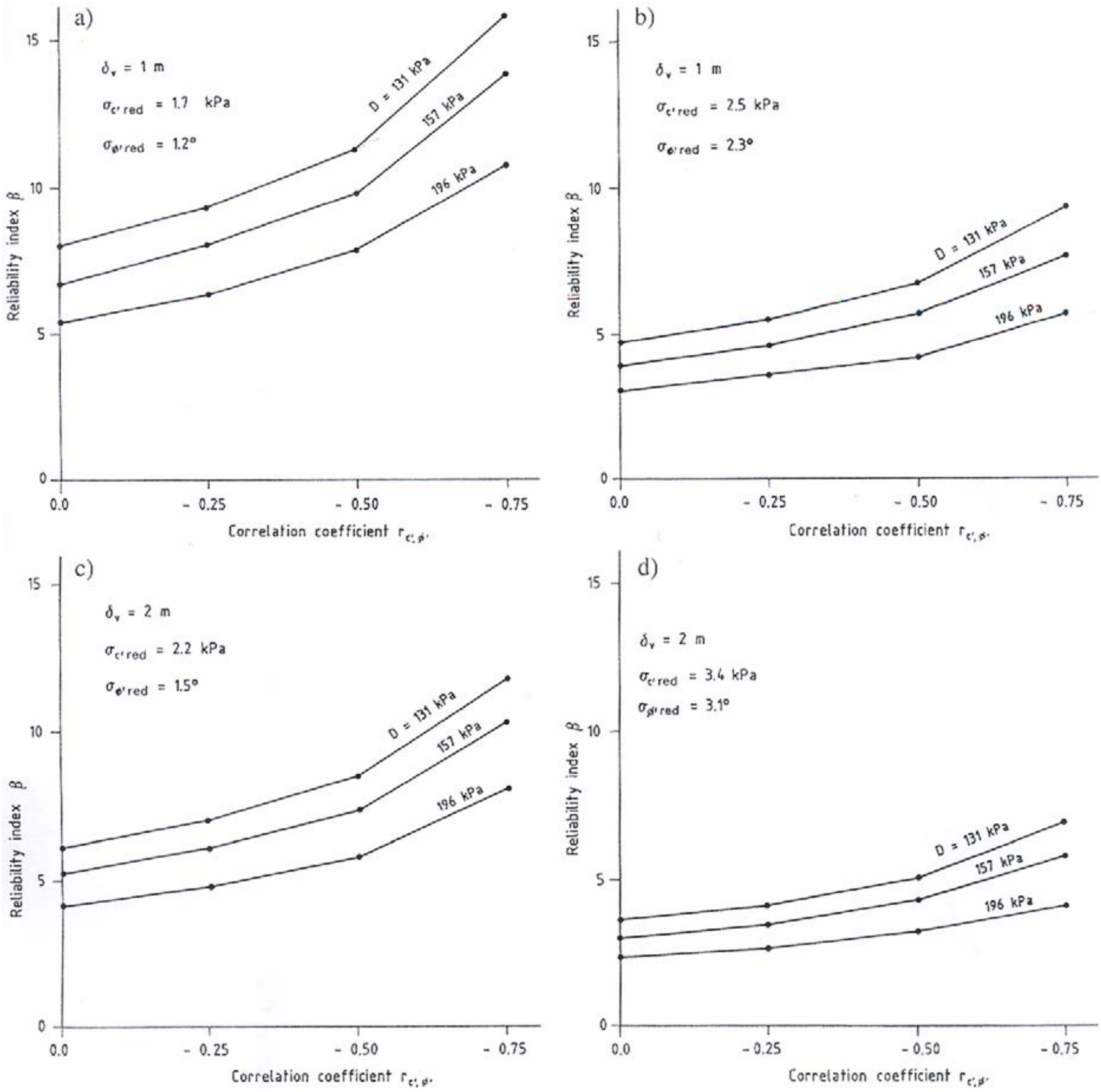


Fig.1 a, b, c, d Reliability Index of shallow foundations for different values of r $_{c,\varphi}$, demand D and fluctuation scale δ (From Cherubini 2000).

In the Performance function or Safety Margin (C-D), the term D (Demand) has been assumed equal to 196 kPa, value correspondent to the bearing capacity calculated through the deterministic approach, assuming as input the mean values of c and φ and a safety factor equal to 2 and assuming F=2.5 and 3 at 157 and 131 kPa.

Moreover it has been considered a possible correlation between c and φ. Some data regarding that (Wolff 1985, Yucemen et al 1973, Lumb 1970, Harr 1987, Cherubini 1997) indicate possible values of rc,φ between -0.24 and -0.70.

On the basis of this literature data in this paper the rc,φ explored range is the one between 0 and -0.75.

The variance reduction is evaluated according to the expression used by Keaveny et al (1989).

The obtained results, reported in Fig. 1a, b, c, d, can be synthesized in this way:

-For the deterministic safety factors of the ultimate bearing capacity F=3, 2.5, 2 the corresponding values of β in soils characterized by cohesion and friction angle, having the indicated coefficients of variations, are sufficiently high. Except for the case (Fig. 1d) in which δ=2 and COV of φ and c are higher.

-When a negative correlation between c and φ is considered in reliability evaluation very different β values are obtained which are very high (especially for r = -075). In other words if the correlation between the two parameters that contribute to strength in the investigated problem is not considered, the evaluation of β is lower than those actually observed when a negative correlation is present.

-The effect of variance reduction is remarkably strong. In particular for δ=1m. β is always 34% higher than it is when δ=2 m.

-Consequently it seems important to stress that δ and r have a significant effect on the results of reliability evaluations whereas an increase of up to 50% in demand seems to have a less important influence.

Fenton and Griffiths (2002) using random field theory and elasto-plastic finite element method analysis evaluate, for soils with spatially varying shear strength, the extent to which spatial variability and cross correlation of soil properties (c and φ) affect reliable bearing capacity.

The study is very complete; here only a brief synthesis is reported.

The analysis is performed for a strip footing placed on soil surface in a two dimensional approach. Soil is assumed to be weightless. On these assumptions the ultimate bearing capacity equation simplifies to:

$$q_{ult} = cN_c \tag{4}$$

and dividing by the cohesion mean μ_c we have:

$$M_c = (c/\mu_c)N_c \tag{5}$$

and consequently

$$q_{ult} = \mu_c M_c \tag{6}$$

The expression of Nc, for homogeneous soil (properties spatially constant):

$$N_c = \frac{e^{\pi \tan\varphi} \tan^2\left(\frac{\pi}{4} + \frac{\varphi}{2}\right) - 1}{\tan\varphi} \tag{7}$$

derives from a failure slip surface being a logarithmic spiral, giving reasonable agreement with some test results (Bowles 1996). In practice it is known that the actual failure surface will be somewhat more complicated essentially for spatial variation in soil properties.

In the study cohesion is assumed to be lognormally distributed a consequently always positive; the friction angle φ is assumed to be bounded by a minimum and a maximum value, resembling a Beta distribution as a simple transformation of a standard random field.

Cohesion and friction angle have similar correlation lengths.

A theoretical model for Mc to define its first two statistical moments, based on geometrical averaging, is proposed by the Authors.

Also, the bearing capacity analysis was performed by means of a finite element code using an elastic perfect plastic stress-strain law with a classical Mohr-Coulomb failure criterion. This finite element method is based on five parameters: E, υ , ψ assumed as constant values , c and φ randomized according to the values reported in Table 1.

For each set of assumed statistical properties of Table 1 Monte Carlo simulations have been performed involving 1000 realizations of soil property random fields.

Table 1 Random field parameters used in the study

Parameter	Value
δ	0.5, 1.0, 2.0, 4.0, 8.0, 50
COV	0.1, 0.2, 0.5, 1.0, 2.0, 5.0
r	-1.0, 0.0, 1.0

Some of the results obtained by the simulations are the following:

-Increasing soil variability the mean bearing capacity factor becomes quite significantly reduced from the traditional case, being less than Prandtl solution based on mean values. The greatest reduction is observed for positively perfectly correlated c and φ ($r_{c,\varphi}$=1) the least for $r_{c,\varphi}$ = -1.

The cross correlation between c and φ seems to have minimal effect on the sample standard deviation of lnMc but is influenced by correlation length. Correlation length on the other side does not have a significant influence on mean value of Mc even if there is a worst case of correlation length of δ= B where B is the width of foundation. Prandtl solution is considered still largely applicable in the case of spatially varying soil properties if geometrically averaged soil properties are considered. Authors state that model error has been neglected in the analysis.

Peschl and Schweiger (2002) have proposed on the basis of the previously synthesized and commented elaborations of Fenton and Griffiths, a practical method that makes use of a standard finite elements code.

In Table 2 are reported the results obtained with FOSM and with three methods PEM (Rosenblueth 1975, 1981, Harr 1989, Zhou and Nowak 1988) considering the case in which COV= 0.5, no correlation between c and φ and δ = 4.0.

The results are reported as a function of the probability of Nc less than 20.7/F, being F a safety factor, assumed equal to 2 and 4.

The obtained results are in quite good agreement with the Random field model but the same one, as put into evidence by Peschl and Schweiger is unsuitable for solving complex practical problems for the computational effort involved.

Babu and Srivastava (2007) make use of the Response Surface Method (RSM) to generate approximate polynomial functions for ultimate bearing capacity (and settlement) of a shallow foundation resting on a cohesive frictional soil for a range of expected variations of input soil parameters.

Table 2 Comparison of the probability of design failure (COV = 0.5, r = 0, δ = 4.0)

Method	$P[N_c<20.7/2]$	$P[N_c<20.7/4]$
Random field method	17%	0.1%
FOSM method	18%	1.3%
PEM by Rosenblueth	8%	0.3%
PEM by Harr	11%	0.6%
PEM by Zhou and Nowak	19%	3.6%

Reliability analysis is then performed to obtain an acceptable value of the allowable bearing pressure and is compared with the results of Monte Carlo simulations.

A strip foundation of width B of 1.5 m. and at a depth Df of 1.0 m is analysed.

The variability of cohesion (μc = 5 kPa COV 12%) and that of friction angle (μφ = 25° COV = 6%) and unit weight (μγ = 17 kN/m3 COV 6%) are considered.

All the variables follow normal distributions and is also considered the possibility for a cross correlation between c, φ and γ according to values of r indicated in Table 3.

Table 3 Correlation coefficient between input soil parameters

	c - φ	c - γ	φ - γ
Case 1	- 0.25	+ 0.25	+ 0.25
Case 2	- 0.50	+ 0.50	+ 0.50
Case 3	- 0.75	+ 0.75	+ 0.75

The results demonstrate the applicability of RSM in the assessment of allowable bearing pressure for shallow foundations resting on a c, φ soil.

The correlation between input soil parameters seems to affect only marginally the Reliability Indexes. This could be connected to a mutual compensation of the effects of a negative correlation between c and φ and of a positive one of γ respectively with c and φ.

Hadj-Hamou et al (1995) reported results from a large experimental program on the evaluation of reliability respect to bearing capacity of footings on a sandy soil using the equation of Terzaghi (1967).

Geomechanical properties of sands are obtained from in situ and laboratory tests showing the mean and standard deviations of φ and γ equal respectively to 35.54° and 16.26 kN/m^3 (mean values) and to 1.75° and 1.45 kN/m^3 (standard deviation).

Two series of load tests have been performed on square footings. The first one on three footings 1.0x1.0m and the second one on four 0.71x0.71m footings. The depth and the corresponding bearing capacity are indicated in Table 4.

The scales of fluctuation are assumed to be 0.5 m in vertical direction and 30 m. in the horizontal one.

Considering as performance function the difference between the bearing capacity obtained from load tests and that obtained from application of Terzaghi equation, the Reliability Index is calculated varying depth Df and coefficient of variation of φ. Variance reduction is also included for comparison.

The results show that increasing Df the value of β decreases the more how much more the variability of the considered parameters is high, with or without the reduction of the variance.

The Authors evidence that this could be a problem in a complex structure when foundations could be placed at different ground levels.

Table 4 Results from load tests

Footing Number	B (m)	D_f (m)	q_u^m (kPa)	SERIE
8	1.00	0.20	868.99	
13	1.00	0.20	887.44	N. 1
23	1.00	0.20	817.75	
4	0.71	0.20	767.88	
5	0.71	0.80	1129.27	N. 2
6	0.71	1.10	1205.76	
7	0.71	1.60	1250.09	

A further reliability analysis for evaluating allowable pressure of a strip footing is presented by Dasaka et al (2005). The foundation considered is 1.2 m. wide and rests on the surface of a cohesionless soil deposit.

Tip resistance qc of a CPT test is used to obtain friction angle values as input parameter in Meyerhof bearing capacity equation, instead of using Schmertmann (1978) method able to calculate the bearing capacity directly from qc values.

Measurements uncertainty, spatial averaging are taken into considerations.

In the following are reported some conclusions of the Authors:

-Transformation models has been identified as the primary factor influencing the degree of variability of design parameter;

-Scale of fluctuation δ and spatial averaging length strongly influence the COV of spatially averaged inherent variability;

-Even though the uncertainties concerning variability measurements of cone tip and transformation model are significant the resulting combined variability of design parameter (angle φ) is less;

-The back calculated Safety factors corresponding to a target reliability index of 3 (assumed as sufficiently reliable) are 7.3 if spatial averaging is considered and 5.5 otherwise (simple approach). These values are higher than those generally adopted in WSD approaches;

-The Authors finally state that higher values of safety factors associated with probabilistic allowable bearing capacities demonstrates the importance of uncertainty studies in geotechnics.

7 PROBABILISTIC LIMIT TOLERABLE SETTLEMENTS

As previously examined, many national codes use for the ultimate limit state statistical methodologies for the evaluation of the characteristic values of the resistance parameters. The partial safety coefficients are then calibrated both on global safety coefficients previously used and on the basis of reliability.

As far as the serviceability limit states SLS are concerned many codes use substantially deterministic approaches in some cases making use of characteristic values of the input parameters as for example the same Eurocode 7.

In this way it is possible to determine the inequality:

$$s \leq \delta \tag{8}$$

where S is the estimated settlement and δ the allowable one. Some tables of reference of the last one have been given especially in the past (Sowers and Sowers 1970) that have just an indicative value because the tolerability of a settlement both absolute and differential is obviously linked to the type of structure and to what is around it.

It has however to be pointed out that either allowable or estimated settlements are affected by numerous uncertainties. Moreover it is well known that in many cases especially in presence of broad foundations with high loads, soft soils etc, the serviceability limit states can be heavier in the design of a foundation in comparison to the ultimate limit state (Becker 1996, Tamaro and Clough 2001).

Different structures have different limits of allowable settlements. The limit tolerable settlements are affecting by type and size of the structure, substructure-superstructure interactions, properties of structural materials and subsurface soils and the rate and uniformity of settlement (Wahls 1981, Burland et al 2001, Zhang et al 2001)

Therefore determining the tolerable limit settlement and the angular distortion beyond which there are significant damages to the structures proves to be a complex problem

In a significant note Zhang and Ng (2005) developed a statistical approach based on the real behavior of a relevant number of foundations of bridges (375) and buildings (300).

After a complex analysis of the available data of absolute settlements and angular distortions, the Authors obtain the probability distributions of the tolerable values pointing out that from this study it is possible to obtain the determination of a deterministic tolerable settlement (and/or angular distortion) using a Safety factor for example =1.5 (Wahls 1994).

It is possible also to determine the characteristic tolerable settlements (and/or angular distortion) assuming a specific probability of exceedance, for example 5%.

In Tables 5, 6, 7 e 8a, 8b are reported respectively a summary of settlement and angular distortion of bridges, the statistics of intolerable and limiting tolerable settlement and angular distortion of bridges and the results of tolerable displacement of bridges obtained from different methods.

Table 5 Summary of settlements of bridges

Settlement interval: mm	All bridges*		Steel bridges		Concrete bridges	
	Tolerable cases	Intolerable cases	Tolerable cases	Intolerable cases	Tolerable cases	Intolerable cases
0-25	52	0	32	0	18	0
26-50	40	2	24	2	13	0
51-100	33	10	23	6	9	2
101-150	1	8	0	7	1	1
151-200	3	5	1	4	2	1
201-250	0	5	0	2	0	2
251-375	2	5	2	4	0	1
376-500	1	4	1	4	0	0
501-1500	0	0	0	0	0	0
	132	39	83	29	43	7

Source: based on data reported by Moulton (1985).

*Bridges include steel, concrete and composite bridges.

Table 6 Summary of angular distortion of bridges

Angular distortion interval	All bridges*		Steel bridges		Concrete bridges	
	Tolerable cases	Intolerable cases	Tolerable cases	Intolerable cases	Tolerable cases	Intolerable cases
0-0.001	43	1	29	1	13	0
0.0011-0.002	36	5	22	5	12	0
0.0021-0.003	32	0	25	0	7	0
0.0031-0.004	14	1	11	1	3	0
0.0041-0.005	10	4	7	3	2	1
0.0051-0.006	2	6	1	5	1	1
0.0061-0.008	2	7	1	4	1	0
0.0081-0.010	1	3	0	2	1	1
0.011-0.020	3	20	2	15	1	2
0.021-0.040	1	8	1	5	0	2
0.041-0.060	0	3	0	1	0	2
0.061-0.080	0	2	0	1	0	0
	144	60	99	43	41	9

Source: based on data reported by Moulton (1985).
*Bridges include steel, concrete and composite bridges.

From this table it is possible to notice how the allowable angular distortion varies in the range 0-0042-0.0055, showing values consistent with the reference allowable distortion of 0.004-0.008 for bridges (Wahls 1994, AASHTO 1997).

The obtained allowable settlement values are approximately 90 mm, which are far greater from the reference allowable settlement of 50 mm (Wahls 1994, AASHTO 1997).

The obtained characteristic settlements for α=0.05 are close to the reference design value of 50 mm. Whereas the obtained angular distortion values are only 0.0017-0.00229 much smaller than the reference allowable value. In tables 5-6 are reported in range the settlements and the angular distortions of bridges for intervals of values and distinguishing between steel and concrete structures.

Table 7 Statistics of intolerable and limiting tolerable settlement and angular distortion of bridges

Statistics	All bridges*		Steel bridges		Concrete bridges	
	Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation
Observed intolerable settlement	183	119	191	129	173	88
Limiting tolerable settlement: mm	135	89	133	72	132	77
Observed intolerable angular distortion	0.0161	0.0155	0.0138	0.0135	0.0232	0.0178
Limiting angular distortion	0.0083	0.0076	0.0063	0.0046	0.0078	0.0044

Table 8a Tolerable displacements for bridges from different methods

Statistics	Settlement: mm			
	Mean of limiting value	Allowable value, mean/FS (FS=1.5)	Characteristic value α =0.05	Reference allowable design value*
All bridges	135	90	42	50
Steel bridges	133	89	51	50

Concrete bridges	132	88	47	50

* Wahls (1994), AASHTO (1997).

Table 8b Tolerable displacements for bridges from different methods

Statistics	Angular distorsion			
	Mean of limiting value	Allowable value, mean/FS (FS=1.5)	Characteristic value, α=0.05	Reference allowable design value*
All bridges	0.0083	0.0055	0.0017	0.004-0.008
Steel bridges	0.0063	0.0042	0.0017	0.004-0.008
Concrete bridges	0.0078	0.0052	0.0029	0.004-0.008

Sivakugan and Johnson (2002, 2004) carried out a statistical analysis predicted by the methods of Terzaghi and Peck (1967) Schmertmann et al (1978), Burland and Burbidge (1985) and Berardi and Lancellotta(1994) based on SPT values characterizing the soil. They showed that the ratio between predicted and actual settlements follows Beta distributions.

Consequently the probability density function of this ratio has been calculated for the four above mentioned settlement prediction methods. Using the Beta distribution parameters four design charts were developed which enable the designer to estimate the probability that the actual settlement in the field will exceed a certain limiting value, fixed in 25 mm for granular soils. The use of the charts is also extended from 15 to 40 mm representing options especially in the case of unconventional structures where different settlement limits may be recommended (Fig. 2 a, b, c, d).

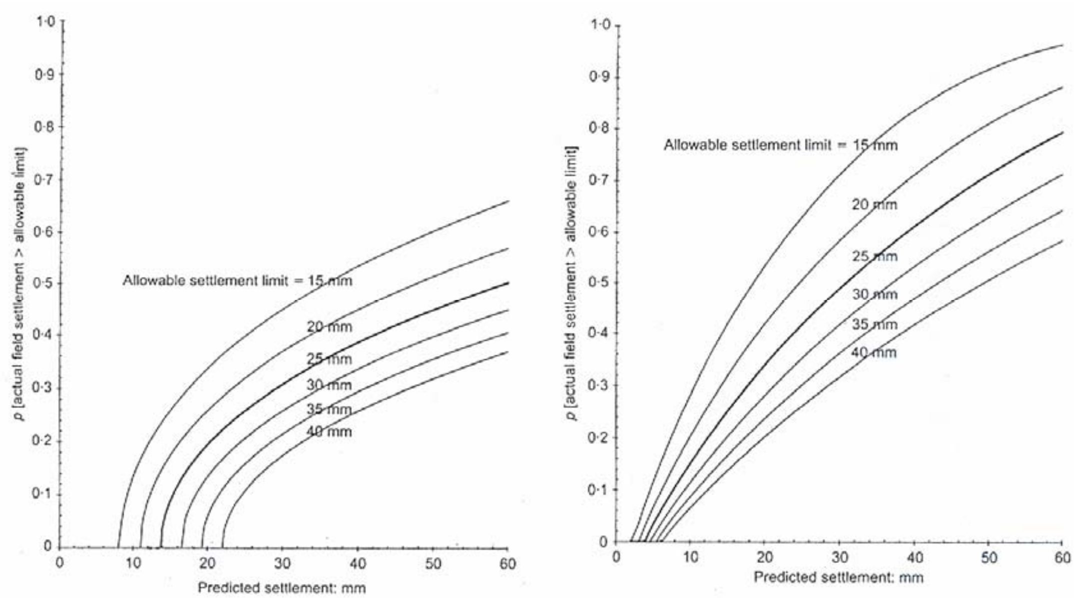


Fig.2 a, b Charts to estimate the probability that actual settlements will exceed a certain limit value (from Sivakugan and Johnson 2003).

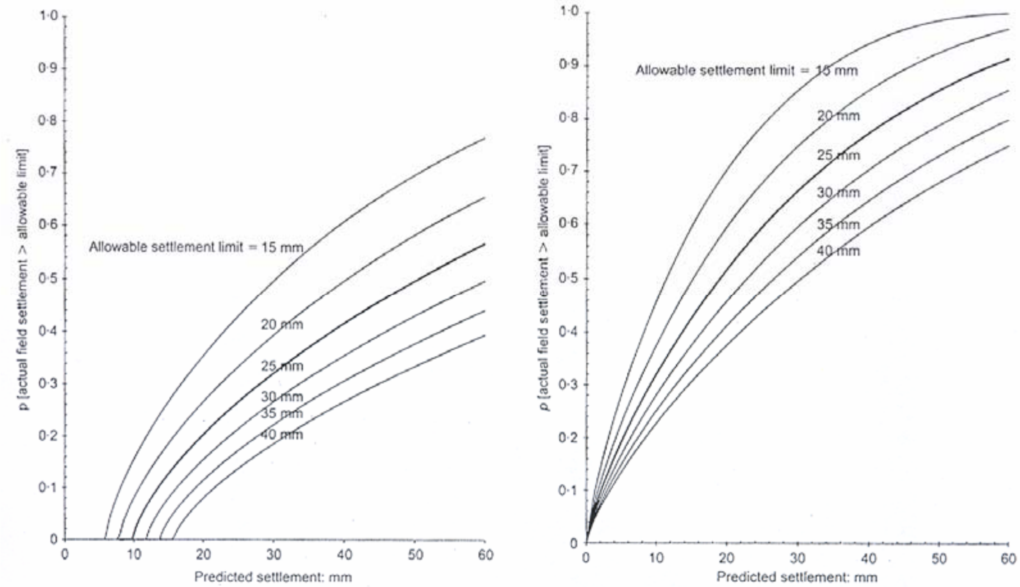
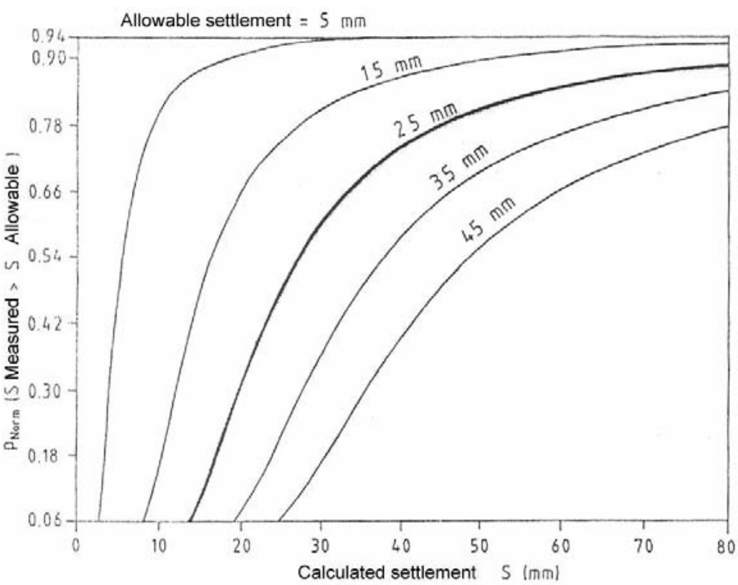


Fig.2 c, d Charts to estimate the probability that actual settlements will exceed a certain limit value (from Sivakugan and Johnson 2003).

Cherubini and Vessia (2005) drawing on the already mentioned work of Sivakugan and Johnson and basing on a previous study (Cherubini and Greco 1998) analyzed 192 cases of calculated and measured settlements of buildingsthroughout the study of the ratio calculated/neasured settlements The analysis was carried out making use of 8 methods based on the results of SPT tests (Terzaghi and Peck 1948, Meyerhof 1965, Meigh and Hobbs 1985, Arnold 1980, Burland and Burbidge 1985, Anagnostopoulos et al 1992 , Schultze and Sherif 1973, Berardi and Lancellotta 1991).

Preliminarily a series of operations were carried out in order to evaluate the precision and accuracy of the 8 proposed formulations. These operations allowed to characterize the Terzaghi and Peck method as the most conservative and the Arnold method as the most precise and accurate. Having preliminarily cleaned the data set from the outliers of the ratio calculated/measured settlement, some abacuses are proposed for the evaluation of the probability of overcoming the settlement from 5 to 50 mm considering not only a Beta distribution but also a normal one, detecting how is it possible to observe from the comparison of the Fig. 3 a, b, that the use of one or of the other frequency distribution doesn't imply particularly significant differences.



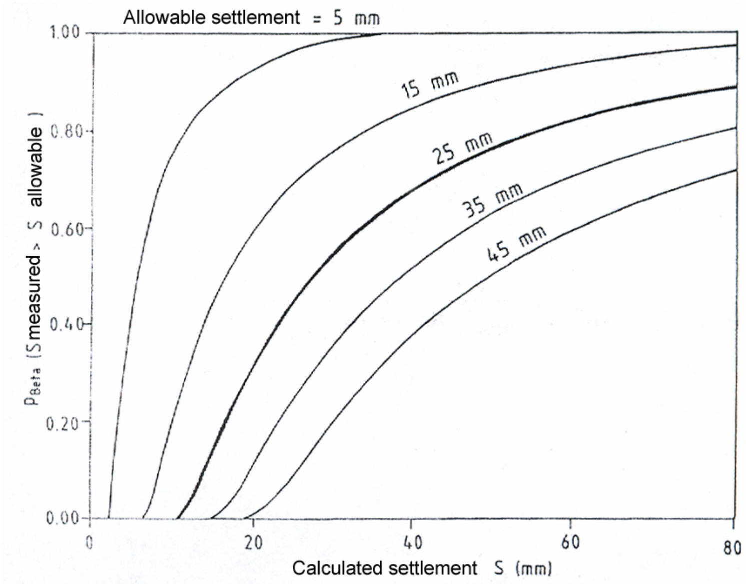


Fig.3 a, b Charts to estimate the probability that actual settlements will exceed a certain limit value (from Cherubini and Vessia, 2005).

8 CONCLUSIONS

The brief synthesis of some recent works on the evaluation of the reliability of foundations in terms of ultimate and of serviceability limit state has allowed drawing some conclusions that will be illustrated in succession:

-The computational models available for the evaluation of the Reliability Index and/or of the probability of determination of a certain limit state are numerous, in some cases even simple to apply and generally give results comparable between each other;

-Some simple expressions of large use (for example three-terms bearing capacity expressions) valid for deterministic design of a foundation on homogeneous soils are not strictly able to understand the behavior of a soil of variable strength properties;

-The most complex and complete often require a highly detailed knowledge of the soil properties through the statistic approach unless a sensitivity or parametric analysis is carried out;

-In this sense even if it appears easy to reach the knowledge of mean trends and variances even on the basis of tables present in the literature, data of major quality and quantity are required for the fluctuation step, the frequency distributions and the eventual correlations between input parameters.

In this context normal and/or lognormal distributions are generally used whereas there is no agreement on the effective truth as far as the correlations between input parameters are concerned;

-More or less significant uncertainties overlap the variability and are linked to a series of factors among which have to be pointed out in a special way, in the case of data coming from tests in situ, those belonging to the relations that transform the read datum in a specified geotechnical parameter;

-Confirmations on real size foundations are available in a significant number as far as the settlements (absolute and differential) are concerned, allowing probabilistic evaluations on statistical basis. The same thing cannot be said as far as the bearing capacity is concerned. For this problem the studies carried out appear generally of parametric type covering even wide intervals as far as the input values of the considered variables are concerned. Few works consider real situations, pointing out once again the difficulties that could be found in such an analysis.

To sum up, the evaluations of reliability of shallow foundations in the geotechnical field appear currently of difficult actuation as they require numerous data, of good quality and examined with adequate procedures as well as many times complex computational models. However they prove to be very useful and probably necessary, to point out the influences of the single parameters in the planning as well as to calibrate the critical points of the codes currently in use.

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