Chapter 6
Effective Stresses and Capillary

6.1 INTRODUCTION

When soils are subjected to external loads due to buildings, embankments or excavations, the state of stress within the soil in the vicinity changes. To study the stability or deformations of the surrounding soil, as a result of the external loads, it is often necessary to know the stresses within the soil mass fairly accurately.

Elastic solutions are quite popular in geotechnical engineering. Here, the entire soil mass is assumed to be a *continuous* elastic media, and the theory of elasticity is applied to determine the state of stress at a point. Some special cases such as the vertical stress increase beneath uniformly loaded square and strip footings are given in most textbooks. Harr (1966), Poulos & Davis (1974), Terzaghi (1943) and several others developed elastic solutions in geotechnical engineering. These developments, with more refinements, have been summarised in design handbooks (Canadian Geotechnical Society 1992; Fang 1991; Winterkorn & Fang 1975).

Nevertheless, soils do not deform elastically. Further, they are particulate media. Therefore, the elastic solutions should only be used with caution.

6.2 EFFECTIVE STRESS CONCEPT

In saturated soils, the normal stress (σ) at any point within the soil mass is shared by the soil grains and the water held within the pores. The component of the normal stress acting on the soil grains, is called *effective stress* or *intergranular stress*, and is generally denoted by σ'. The remainder, the normal stress acting on the pore water, is knows as *pore water pressure* or *neutral stress*, and is denoted by **u**. Thus, the total stress at any point within the soil mass can be written as:

$$\sigma = \sigma' + u \qquad (6.1)$$

This applies to normal stresses in all directions at any point within the soil mass. In a dry soil, there is no pore water pressure and the total stress is the same as effective stress. Water cannot carry any shear stress, and therefore the shear stress in a soil element is carried by the soil grains only.

6.4 VERTICAL NORMAL STRESSES DUE TO OVERBURDEN

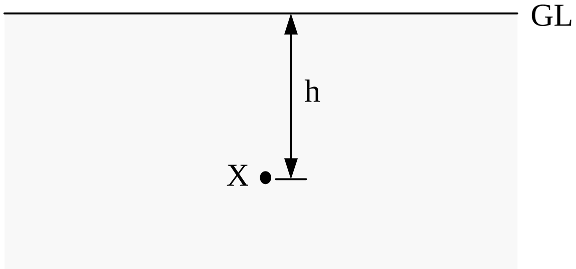

Figure 6.1 Overburden stress at a point in a homogeneous soil

In a dry soil mass having a unit weight of γ (see Fig. 6.1), the *normal vertical stress* at a depth of **h** is simply γh. If there is a uniform surcharge **q** placed at the ground level, this stress becomes $\gamma h+q$.

In a soil mass with three different soil layers as shown in Fig. 6.2, the vertical normal stress at X is $\gamma_1 h_1 + \gamma_2 h_2 + \gamma_3 h_3$.

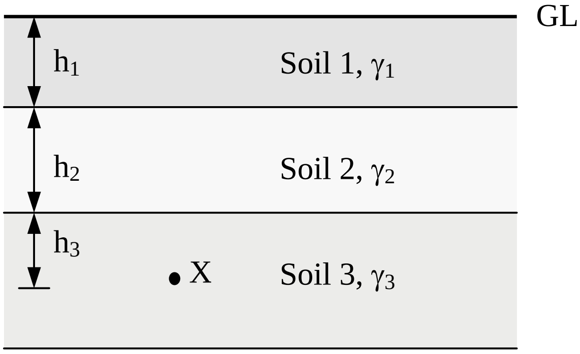

Figure 6.2. Overburden stress at a point in a layered soil

Now let's see what happens in a saturated soil? For the soil shown in Fig. 6.3, for simplicity we will assume that the water table is at the ground level. Let the saturated unit weight and submerged unit of the soil be γ_{sat} and γ' respectively. The *total vertical normal stress* at X is given by:

$$\sigma_v = \gamma_{sat} h \qquad (6.2)$$

The pore water pressure at this point is simply,

$$u = \gamma_w h \qquad (6.3)$$

Therefore, the *effective vertical normal stress* is,

$$\sigma_v' = \sigma_v - u$$
$$= \gamma_{sat} h - \gamma_w h = \gamma' h$$

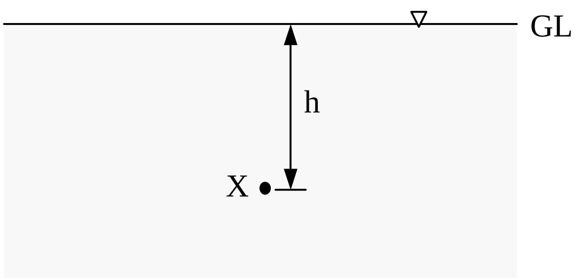

Figure 6.3 Overburden stress at a point in a homogeneous saturated soil

When the water table is at some depth below the ground level as shown in Fig. 6.4, the total and effective vertical stresses and the pore water pressure can be written as:

$$\sigma_v = \gamma_m h_1 + \gamma_{sat} h_2$$
$$u = \gamma_w h_2$$
$$\sigma_v^{'} = \gamma_m h_1 + \gamma' h_2$$

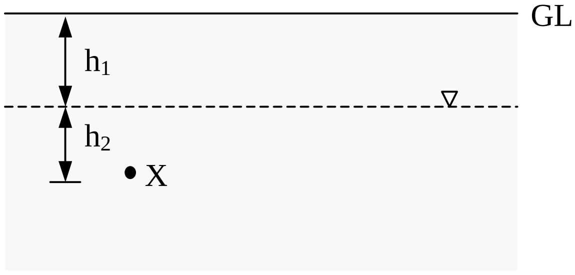

Figure 6.4 Overburden stresses at a point when the water table is below the ground level

When computing total vertical stress, use saturated unit weight for soil below the water table and bulk or dry unit weight for soil above water table.

When the soils are partially saturated, the situation is a bit more complex. Here, the normal stress within a soil element is carried by the water, air, and the soil grains. Therefore, the normal stress can be split into three components and written as:

$$\sigma = \sigma' + \chi u_w + (1 - \chi) u_a \qquad (6.4)$$

Here u_w and u_a are the pore water pressure and pore air pressure respectively, and χ is a constant that can be determined from triaxial test and varies between 0 and 1. In dry soils χ=0 and in saturated soils χ=1.

6.3 CAPILLARY EFFECTS IN SOILS

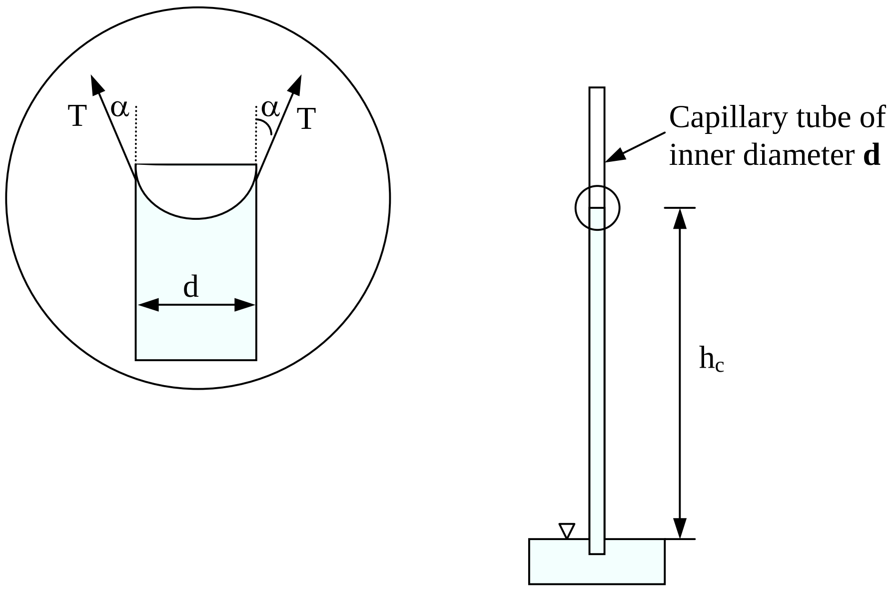

Figure 6.5 Capillary tube in water

Let's look at some simple physics on capillary. A capillary tube is placed in a dish containing water as shown in Fig. 6.5. Immediately, water rises to a height of h_c within the tube.

The water column is held by the surface tension **T** at the top (see inset), which acts at an angle of α to vertical. For equilibrium of the water column, the weight of the water column is balanced by the vertical components of the surface tension. This can be written as:

$$\frac{\pi d^2}{4} h_c \gamma_w = T \cos\alpha \, \pi \mathrm{d} \qquad (6.5)$$

Therefore,

$$h_c = \frac{4T\cos\alpha}{\gamma_{\mathrm{w}} d} \qquad (6.6)$$

Using typical values of T = 0.073 N/m, $\alpha = 0^\circ$ and γ_w = 9810 N/m^3 in Eq. 6.6, it can be shown that:

$$h_c(m) \approx \frac{0.03}{d(mm)} \qquad (6.7)$$

What do these have to do with soils? The interconnected voids within the soil can act like capillary tubes (not straight though) and allow the water to rise well above the water table. The "capillary tube" diamater of a soil is approximately 1/5 of D_{10}. Therefore, the capillary rise within a soil can be written as:

$$h_c(m) \approx \frac{0.15}{D_{10}(mm)} \qquad (6.8)$$

As you would expect, finer the soil, smaller the capillary tube diameter, and larger the capillary rise. This can be also inferred from Eq. 6.8, which works well for sands and silts. Gravels are so coarse that there will be negligible capillary effects. Clays have the most capillary rise. Capillary rise can be few milli metres in sands to several metres in clays. Capillary pressure is a pore water pressure that is always negative. Since this occurs while there is no change in total stress, it increases the effective stresses significantly.

Capillary pressure is always negative and gives a suction effect, and increases the effective stress.

You have to be quite clear with the effective stress principle, which will come in any time we have to compute stresses when there is water in the soil.

EXAMPLE

1. Plot the variation of total and effective vertical stresses, and pore water pressure with depth for the soil profile shown below in Fig. 6.6.

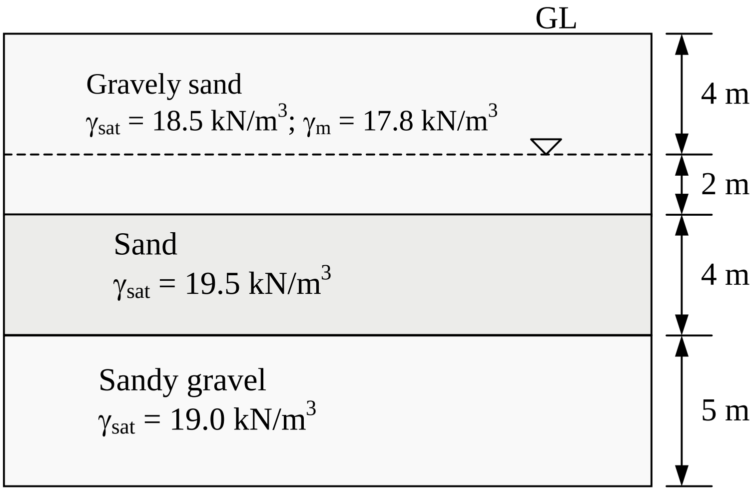

Figure 6.6 Soil profile for Problem 6.1

Solution:
Within a soil layer, the unit weight is constant, and therefore the stresses vary linearly. Therefore, it is adequate if we compute the values at the layer interfaces and water table location, and join them by straight lines.

At the ground level,

$\sigma_v = 0$; $\sigma_v' = 0$; and $u=0$

At 4 m depth,

$\sigma_v = (4)(17.8) = 71.2$ kPa; $u = 0$

$\therefore \sigma_v' = 71.2$ kPa

At 6 m depth,

$\sigma_v = (4)(17.8) + (2)(18.5) = 108.2$ kPa

$u = (2)(9.81) = 19.6$ kPa

$\therefore \sigma_v' = 108.2 - 19.6 = 88.6$ kPa

At 10 m depth,

$\sigma_v = (4)(17.8) + (2)(18.5) + (4)(19.5) = 186.2$ kPa

$u = (6)(9.81) = 58.9$ kPa

$\therefore \sigma_v' = 186.2 - 58.9 = 127.3$ kPa

At 15 m depth,

$\sigma_v = (4)(17.8) + (2)(18.5) + (4)(19.5) + (5)(19.0) = 281.2$ kPa

$u = (11)(9.81) = 107.9$ kPa

$\therefore \sigma_v' = 281.2 - 107.9 = 173.3$ kPa

The values of σ_v, u and σ_v' computed above are summarized in Table 6.1.

Table 6.1 Values of σ_v, u and σ_v' in Ex. 1

depth (m)	σ_v (kPa)	u (kPa)	σ_v' (kPa)
0	0	0	0
4	71.2	0	71.2
6	108.2	19.6	88.6
10	186.2	58.9	127.3
15	281.2	107.9	173.3

The plot is shown below in Fig. 6.7.

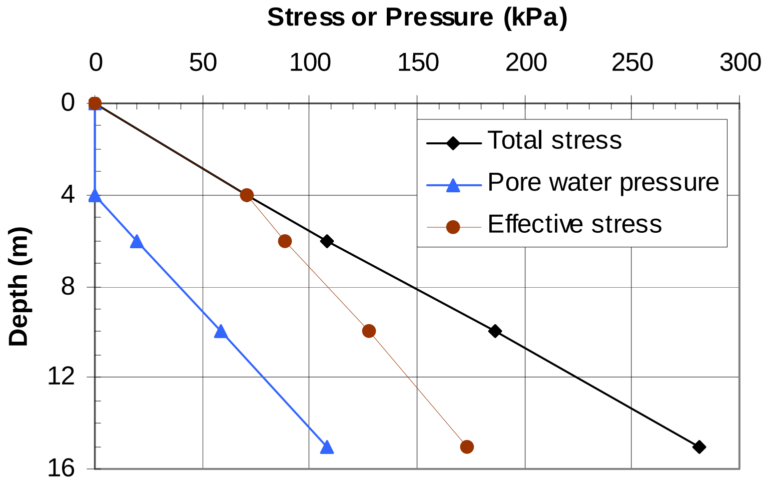

Figure 6.7 Variation of σ_v, u and σ_v' with depth

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